Bipolar Change

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Introduction

In this paper I reconsider the natural characterization of change and non-change that arises from the algebraic approach: this sees change as yang in contrast to non-change, which is yin. Following a persuasive example from Alain Stalder, rather than consider change solely in contrast to non-change, I develop a formal characterization of different forms of change considered relative to each other. This extension allows the internal structure of a change to be made explicit in a new way, bifurcating the change into yang parts and yin parts. I call this extended definition of change *bipolar change*.

Now, some forms of change might be properly considered as yin, for example change arising from the inherent energies of a situation; whilst other changes would be yang, for example change imposed from outside a situation. However, most previous work has been restricted to representing change as entirely yang against a background of yin non-change (*e.g.* Goldenberg 1975, Schöter 1998). Clearly this is a significant limitation. The work presented in this paper allows the simultaneous representation of both yin and yang aspects of change within the algebraic language. This, in turn, allows more complex forms of change to be explored.

I develop a number of extended examples showing how this idea can be applied in detail. I start by showing how the total change can be divided into particular polar domains such as inner and outer, or Heaven and Earth. This general technique is then applied to analyse change in various ways: i) through lattice direction, ii) through change of correctness and iii) through change of correspondence. Finally, the particular examples are then placed in a broader context by considering their general recursive expansion. All of these forms of change conform to a minimality constraint, we could call them "classical". However, bipolar expansion opens up a possibility beyond minimality, a form of dynamic non-change. When considered from the information theoretic perspective on change, this is shown to be conceptually similar to vacuum polarization in quantum theory.

Although the material in this paper is necessarily mathematical in nature, the philosophical implications are important. The Yijing provides a symbolic language for describing patterns of change: situations are described by structured groups of hexagrams, and the connections within a structure determine the relationships in the interpretation of the situation. The algebraic analysis of the symbols gives rise to formal mathematical descriptions of the structures of change, which allows the internal dynamics of its patterns to be explored. This opens up a rich realm of new interpretive possibilities. For the mathematician, the algebraic definitions speak for themselves and carry the true meaning of the work. However, I appreciate that this

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aspect is not easily accessible to the non-specialist reader. I have therefore tried to make the implications as clear as possible throughout.

Some proto-mathematical properties of the symbols were clearly significant to the early masters of change, particularly those engaged in study of the Image and Number tradition. That tradition characterized a number of important properties of the symbols, in particular correctness and correspondence, which, from the modern perspective, have clear mathematical representations.[1] The material presented in this paper allows us to incorporate consideration of these properties into the contemporary algebraic analysis of change, thereby demonstrating that although these properties were not originally formulated in terms of precise, algebraic language, the methods of the Image and Number sages find a ready home in this contemporary analysis.

The development of bipolar change presented in this paper unavoidably leans extensively on the formal techniques that I have used in my previous work. The material in “Boolean Algebra and the Yijing” (Schöter, 1998) provides the background formalism, and “Correctness and Correspondence” (Schöter, 1999) and “Flowers and Steps in the Boolean Lattice of Hexagrams” (Schöter, 2004) provide the material from which the examples are drawn. Therefore, before venturing on to bipolar change itself, I shall review the necessary preliminaries. Those readers familiar with this material should skim the first section to reacquaint themselves with the notation and then move on to the section describing Normal Algebraic Change.

Boolean Algebra, Correctness and Correspondence

In “Boolean Algebra and the Yijing” (Schöter 1998) I explore how formal algebraic techniques can be applied to the analysis of change. The following section presents a brief outline of the basic notation and its semantics.

Boolean Algebra

The second proposition of Chapter 3 of Boole’s *Laws of Thought* (1854, p47) is that the goal of his logical system is “to determine the logical value and significance of the symbols 0 and 1”. The success of this project means that Boolean algebra provides us with a complete formal language for describing the interactions between patterns of 1s and 0s. In the context of the Yijing, we interpret these values as yin for 0 and yang for 1. The symbols of the Yi consist stacks, or vectors, of independent values, taking stacks of three values to form trigrams and stacks of six to form hexagrams. The algebra then provides connectives for transforming one symbol into another, and for combining pairs of symbols into a third. Using these operations we can explore the formal patterns of interaction for yin and yang. Let’s begin by describing three standard algebraic operations. Table 1 shows their definitions.

[1] Bent Nielsen’s excellent work *A Companion to Yi Jing Numerology and Cosmology* (Nielsen 2003) provides a detail survey of these ideas.

A	~*A*
0	1
1	0

v		*B* 0	*B* 1
A	0	0	1
	1	1	1

&		*B* 0	*B* 1
A	0	0	0
	1	0	1

Table 1: The Basic Boolean Operators

Consider the first operator: in this case ~*A* means the opposite, or complement, of *A*. The table shows that 0 becomes 1 and 1 becomes 0; that is, yin becomes yang and yang becomes yin. When this operator is applied to hexagrams, the operator is applied to each line in turn. For example, ~䷶ = ䷺. This operation gives pairs of hexagrams traditionally known as *pang tong gua* or "laterally linked hexagrams" (see Nielsen 2003, p185).

The other two operators take two values and combine them to generate a third value. The first of these is known informally as "or", technically called *disjunction* or *least upper bound*. The table shows its effect – if either of the input values are 1 (yang) then the result is also 1 (yang); only if both input values are yin is the result yin. We could therefore say that the "or" operation combines the yang energy from its inputs. We can apply the operator to hexagrams, line by line; for example, ䷯ v ䷶ = ䷪. The final operator shown in the tables above is informally called "and", technically known as *conjunction* or *greatest lower bound*. The table shows its effect – the output is yang only if both of the input values are yang, otherwise, if either value is yin, the result is yin. Therefore, this operator combines the yin energy from its inputs. For an example with hexagrams we can see that ䷯ & ䷶ = ䷎. Neither of these two operations seems to have been used in the classical apparatus of the Yijing.

The other Boolean operator used extensively in this work is called "xor", meaning "exclusive or". One way to understand this operator is to see it as determining the difference between its two inputs. Consider the following table:

x		*B* 0	*B* 1
A	0	0	1
	1	1	0

Table 2: The Xor Operator

This shows that the result of the xor operator is yang when *A* and *B* are different, otherwise the result is yin. We can consider how this operator applies to hexagrams with an example: ䷯ x ䷶ = ䷹. This operator is very versatile, and it can be applied in many different ways in the analysis of change.

Lattice Structures

The algebra is intimately connected to a partial ordering, written $X \leq Y$, which in turn gives rise to structures known as lattices (for more details on this see Schöter 2004). For the basic symbols, we have $0 \leq 1$, and this then extends to larger symbols, line by line so, for example, ䷾ ≤ ䷤, but not ䷾ ≤ ䷕. This ordering is not merely a matter of the amount of yang in the symbol, but also the pattern of its distribution. From the perspective of the algebraic connectives, where *X* and *Y* are arbitrary symbols:

(1) If $X \leq Y$, then $\sim Y \leq \sim X$.

(2) *X* ≤ *X* ∨ *Y*

(3) *X* & *Y* ≤ *X*

These connectives then give rise to structures, between a maximal and a minimal element, such that for every element *X* in the structure, *X* ≤ *MAX* and *MIN* ≤ *X*. If we take the following values:

(4) *MAX* = ䷝

(5) *MIN* = ䷽

This gives the structure shown in Figure 1. This is a two-dimensional lattice. The number of lines different between MAX and MIN determines the dimensionality of the resulting lattice. In this case, two lines are different.[2]

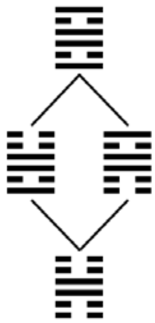

Figure 1: The Lattice Structure for 987789

If we have two arbitrary hexagrams *X* and *Y*, then the limits of the smallest structure that contains them both is given by:

(6) *MAX* = *X* ∨ *Y*

(7) *MIN* = *X* & *Y*

Remember "or" is the combination of yang energy and "and" is the combination of yin energy. This helps to understand how equations (6) and (7) build the lattice. Below, we will see how these lattice structures are used to represent change, and we will analyse the path of a change through the structure.

Correctness

In "Correctness and Correspondence" (Schöter 1999) I show how these traditional notions can be defined within the formal context of Boolean algebra. This material is relevant to the examples of bipolar change explored below. Here I shall restrict my discussion to developing the necessary formal definitions. For more general details of correctness and correspondence, the reader is referred to Wilhelm (1983, pp360–362).

In brief, correctness shows the appropriateness of a yin line or a yang line, depending on its placement in the symbol. Odd numbered lines are correct when yang, and even

[2] I use the standard numerical notation for lines: 6 is a changing yin line, 7 is a stable yang line, 8 is a stable yin line, and 9 is a changing yang line.

numbered lines are correct when yin, so the completely correct hexagram is ䷾ and the completely incorrect hexagram is ䷿.

We shall define the correctness of a hexagram *H* in terms of another hexagram *C* which has a yang line wherever a line in *H* is correct and a yin line wherever a line in *H* is incorrect (see Schöter 1999, p30). First, consider that every line in ䷿ is incorrect. This gives us the key to define the function c(*H*):

(8) c(*H*) = *H* x ䷿

Here, the xor operator proves its worth. One way to view the correctness of a hexagram is to see it as the difference between that hexagram and absolute incorrectness. This is what equation (8) captures in a formal statement. Note that this only works for hexagrams because the function makes reference to a hexagram in its definition.

We can apply this definition to some examples to clarify its operation. Firstly, c(䷾) = ䷀, because every line in ䷾ is correct, the result of function c(.) is all yang lines. Next, consider c(䷁) = ䷿, here, the second, fourth and sixth lines of ䷁ are correct, so these are the lines that are yang in the result. Finally, c(䷀) = ䷾. In this case, we know that the second and fourth lines of ䷀ are incorrect and, appropriately, these lines are yin in the result of the function.

Correspondence and Resonance

The matching lines of the upper and lower trigrams are said to correspond. That is, the first and fourth lines of the hexagram correspond, the second and fifth lines correspond, and the third and sixth lines correspond. Here, I shall say that when a pair of corresponding lines have different polarities, they resonate; when they have the same polarity, there is no resonance.[3]

Before we can develop a formal definition of this idea, we need to provide an auxiliary structural operation. To this end, I introduce the following notation:

(9) *G* = e(*H*)

This means that hexagram *G* is the result of exchanging the trigrams in hexagram *H*. For example, ䷯ = e(䷺). Nielsen gives the Chinese term for this operation as *liang xiang yi* (2003, p154), meaning the exchange of the two images.

The internal workings of this operation need not detain us here. This simple notation lets us define the function r(*H*), meaning the resonance in *H*.[4]

[3] The use of the term *resonance* for this relationship comes from Richard Smith (1998, p41), who gives the Chinese term for this relationship as *zheng ying*, which he translates as "correctly resonates". This term captures the energetic nature of the relationship.

[4] Note that the definition used here is slightly different from that put forward in Schöter 1999 (p32), where the result of r(*H*) is a trigram rather than a hexagram. Defining r(*H*) to generate a hexagram instead of a trigram makes it possible to apply it more generally, to analyse the internal structure of change.

(10) r(*H*) = *H* x e(*H*)

This says that the resonance of a hexagram *H* is equal to the difference between *H* itself and the trigram exchange of *H*.

Consider some examples – firstly:

(11) r(䷩) = ䷩ x e(䷩)
= ䷩ x ䷟
= ䷀

This shows that in a hexagram composed of complementary trigrams, all the lines are resonant. In the subsequent examples, I shall omit the intermediate stages and only present the result. So, next, consider r(䷲) = ䷁, showing that in a hexagram composed from a doubled trigram, no lines are resonant. Finally, r(䷛) = ䷝ shows that the first and fourth lines and the third and sixth lines are resonant, whilst the second and fifth lines are not resonant.

Normal Algebraic Change

This concludes the necessary preliminaries. We shall proceed by reviewing the standard algebraic definition of change, where change is represented as a yang happening against a yin background of non-change.

The Standard Definition

The normal algebraic description of change was first explored by Goldenberg in the context of group theory (Goldenberg 1975) and then independently rediscovered by myself in the context of Boolean algebra (Schöter 1998). Its definition is given by the following equation:[5]

(12) *P* x *Q* = *R*

Here *P*, *Q* and *R* are usually taken to be hexagrams,[6] such that *P* is the principle state under consideration and *R* is some related state. The description of the relation between those states is given by a unique third state *Q*, describing the quanta of change. The equation says that the difference between *P* and *Q* gives *R*; the principle state xored with the quanta of change gives the related state. The technique resulting from this application of Boolean algebra is that wherever a line in *Q* is yang, *R* is different to *P*, and wherever a line in *Q* is yin, *R* is the same as *P*. Goldenberg (1975, p163) shows that this operation gives a unique value for *Q* for each possible pair of values for *P* and *R*.

[5] Goldenberg uses the ⊕ symbol for this operation, defining it as addition, modulo 2; the notation is different, but the formal properties are the same.

[6] In fact, the definition is completely general and they could be symbols with any number of lines.

Philosophical Considerations

Let us consider change in relation to non-change. Non-change is the static background against which the dynamics of change can be perceived; Wilhelm makes this explicit (Wilhelm 1983, p280) and Cheng clarifies and extends the idea (Cheng 2005). If we are to relate these two terms on a polar axis, then it seems unproblematic that, *in relation to each other*, non-change should be yin and change should be yang. This provides a formal representation of the common philosophical position expressed by the two authors which, in turn, fits with the "standard" algebraic description.

We noted that one of the features of the definition given by equation (12), above, is that the algebra represents the changing lines in *P* by yang lines in *Q* and the unchanging lines in *P* with yin lines in *Q*. Thus, the algebra is in agreement with the basic philosophical consideration. For example, consider the change described in traditional numerical terms as 769687. This has changing lines in the second, third and fourth places, and its algebraic representation, as a specific example of equation (12), would be:

(13) ䷕ x ䷟ = ䷥

So, we see that *Q* has yang lines in the second, third and fourth places, corresponding to the changing lines in *P*, whilst the remaining lines in *Q* are yin, corresponding to unchanging lines in *P*.

The physicist Alain Stalder (personal communication) has raised the question as to whether all change really is yang. In relation to non-change, I maintain the position outlined above, that change is yang and non-change is yin. However, if we consider the relationships between different kinds of change, rather than the relationship between change and non-change, then we quickly see indeed, that some forms of change are yang relative to other forms of change, which are yin. Stalder's example (see his web site) is derived from the division of experience into internal and external: things in the outside world tend to be seen as stable, and energy needs to be applied to create change, thus change in the outside world is yang; however, in the inner world, in the mind, things tend to be in flux, and energy needs to be applied to restrain change, thus change in the inner world is yin. These conceptual possibilities have lead me to consider whether it is possible to extend the standard algebraic definition given as equation (12) to include this relative bifurcation of change.

Further, both Wilhelm and Cheng posit a variety of forms of change which contrast with the base of non-change. This provides additional support for the idea that some extension to the standard algebraic representation is required. For Wilhelm, the characterisation of the forms of change is essentially geometric: change is either linear or cyclic. In Cheng's system the varieties of forms of change have a more complex description, defined in terms of characteristics to which all forms of change should conform. I shall return to these additional ideas, considering the extent to which they can be incorporated into bipolar change, in the conclusion.

Bipolar Change

In the light of the considerations outlined above, I now introduce the notion of bipolar change, and provide an algebraic description of its properties.

The Algebraic Definition

In the most general case, we want to allow that some aspects of a change may be yin and some may be yang. However, we do not want to disturb the existing algebraic definition of change, as this has proven to be an extremely successful technique. Because Q is the quanta of change, this is where we need to describe any yin/yang distinctions within the change, and to do this we need to break Q into two components. The xor operator shows its versatility again by providing us with the best way to do this.[7] Equation (14) gives the definition:

(14) $Q = X \times {\sim}Y$

This says that Q can be seen as the symmetric difference of X, the yang component of change and the complement of Y, the yin component of change. This definition is formulated by consideration of the yin/yang nature of the relationship between the two terms: once X is taken as the yang component, we use the Boolean negation operation to indicate Y as the yin component. However, because of the symmetric nature of the difference operation x, this definition is actually equivalent to both of the following equations: $Q = {\sim}X \times Y$ and $Q = {\sim}(X \times Y)$. These equivalences arise directly as a consequence of the properties of Boolean algebra, and they show that the negation can appear anywhere in the bipolar expansion. The form given in (14) is chosen as it is the clearest representation of the intended concept.

Note that Q itself remains unchanged, so the definition of normal algebraic change remains applicable. However, we can now look inside Q and consider different aspects of change relative to each other. If we combine the normal definition of change (12) with this bipolar extension (14), we get the following equation:

(15) $P \times (X \times {\sim}Y) = R$

Before we explore how this definition works in practice, let's consider some different ways in which change can be bifurcated, as this will give us the examples to justify the additional complication of bipolar change.

Characterizing the Bifurcations Linguistically

One suggestion for an interpretation of bipolar change is to consider the origin of the aspects of the change: these could be either an imposition from outwith the situation, hence yang in X, or as intrinsic to the natural energy of situation, hence yin in Y. We also considered the idea that change in different domains would have different natures: Stalder suggests that outer change is yang and inner change is yin. There are other ways we might provide a bipolar characterization of the different forms of change.

We could consider the division in terms of the direction of the change, up or down through the lattice structure. If the change from P to R is from yin to yang, this is a

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[7] We could consider other possible ways of decomposing Q, but following Goldenberg's result we know that xor gives us unique pairs of X and Y, whilst other decompositions would not. For example, if we took $Q = X \vee {\sim}Y$, then for any particular X we could have a range of possible values for Y. Although we will see there is some possible variation for X or Y relative to Q, we wish to restrict their covariance so that once a value for one is known, the value of the other is determined.

movement up the lattice, a yang change in X. If it is a change from yang to yin, this is a movement down the lattice, a yin change in Y. Alternatively, we could consider the effect that the change would have on the correctness of the lines. A change from incorrect in P to correct in R would be a yang change in X, whilst a change from correct in P to incorrect in R would be a yin change in Y. Or again, consider the effect of line changes on how the resonance within P compares to the resonance within R – the possibilities here are more complex. If a line changes, then its resonance in P may be different to its resonance in R, or it may be the same, depending on other changes in P; if its resonance changes, then it either increases or decreases in R. Thus, there are more possibilities to consider, and we will have to delve deeper into the internal structure of Q.

We can ask: what are the relationships within Q, between X and Y, that correspond to the different alternatives discussed above? Are there useful constraints that we can consider? Do the constraints help us select from the alternative interpretations? Are existing forms of change preserved and further contextualized? Are new forms of change revealed? We see below that the answer to all of these questions is yes.

Constraining *X* and *Y*

Let's start by thinking about the possible constraints and relationships that we can uncover. Firstly, consider the possible extremes of Q itself, and its effect on X and Y:

(16) If there is no change in the situation, then we know that $Q =$ ䷁. It follows from this that $X = \sim Y$.

(17) Conversely, if everything changes, then $Q =$ ䷀. It then follows that $X = Y$.

Equation (16) tells us that if there is no change, then $Q =$ ䷁, and the two components, X and Y, must combine together to cancel each other out. This follows from equation (14) because we know that when there is no change, $X \times \sim Y =$ ䷁. From this we get $X =$ ䷁ $\times \sim Y$, and therefore $X = \sim Y$. Equation (17) is similar – this says that the components of change, X and Y, are identical when everything changes. In this case, X and Y must combine together so that $X \times \sim Y =$ ䷀. Therefore $X =$ ䷀ $\times \sim Y$, and via a standard theorem of Boolean algebra, therefore $X = Y$.

Now let's consider the possible extreme values for X and Y and their effect on Q.

(18) If $Y =$ ䷀, then $Q = X$. All change is yang, and bipolar change collapses to normal algebraic change; X shows every change through yang energy.

(19) If $X =$ ䷁, then $Q = \sim Y$. All change is yin, and bipolar change collapses to its opposite representation; Y shows every change through yin space.

These two expressions describe how X and Y behave when change is entirely yang or entirely yin. When change is entirely yang, as described in (18), then no part of the change is yin, and only X carries information about the change. Hence, X and Y collapse to the standard definition, given as (12). When change is entirely yin, as described in (19), then no part of the change is yang, and only Y carries information.

In that case *X* and *Y* collapse to the definition used by Karcher (1997, pp26–27).[8] This yin form of analysis actually seems to have first been suggested in 513BCE when a scribe called Zaimo refers to the first line of the hexagram Creative ䷀ as its Coming to Meet (䷫) line, its second line as its Fellowship with Men (䷌) line and so on. This is cited in de Fancourt (1997, p41).

With any values for the components of change, other than those described in (16)–(19), the definition of bipolarity (14) gives *X* and *Y* some intermediate, covariant values taken from the remaining range of hexagrams. So, let's continue by looking at some relational constraints, starting with the most natural:

(20) $X \leq Q$

This says that there's no change in *X* that's not in *Q*. If this constraint is imposed, then we also automatically get:

(21) $\sim Y \leq Q$

Which says there's no change in *Y* that's not in *Q*. Let's call this the *minimallity constraint* because *X* and *Y* are minimal with respect to *Q*. This constraint ensures that there is no change other than that specified directly in *Q* and, in the process, it greatly reduces the range of values for *X* and *Y*. An example will help clarify the operation of these constraints.

An Initial Example

Consider our example again:

(13) ䷕ x ䷟ = ䷙

From the definition of bipolarity (14) we then get:

(22) ䷟ = X x $\sim Y$

With the minimality constraint (20) in place, we can enumerate all the possible values pairs for *X* and *Y*:

(23) from (13) via (20) and (22)

X	䷁	䷏	䷎	䷽	䷆	䷧	䷭	䷟
Y	䷩	䷘	䷤	䷌	䷼	䷉	䷈	䷀

In this example we see that (18) and (19) above hold: when *X* = ䷁, then *Y* shows every change through yin space ䷩, and $Q = \sim Y$; similarly, when *Y* = ䷀, then *X* shows

[8] Since our joint work in Karcher and Schöter (2002), Karcher has actually taken to using both yin and yang representations of change (see, for example, Karcher 2003, pp45–46). Here I wish to suggest that both forms of change are not usefully considered as acting together simultaneously. The expressions given as (18) and (19) make it clear that pure yang change and pure yin change are two ends of a spectrum. As I develop the examples below, it will be seen that simultaneous yin change and yang change can occur, but only between these extremes.

every change through yang energy ䷟, and $Q = X$. Then, in between, X and Y take covariant values, with the range of pairs limited by the minimallity constraint (20), so both $X \leq$ ䷟ and $\sim Y \leq$ ䷟ hold for all the pairs of values in the table.

The question we shall now consider is, which of the forms of bipolar change so far discussed correspond to which X/Y pairs above?

Characterizing the Bifurcations Algebraically

If we want to step away from normal algebraic change, into the complexity of bipolar change, how can we decide between the wealth of covariant possibilities that arise? Above, we suggested some possible interpretations for bipolar change; let’s now return to look at those.

Specified Division

One idea was that the changes in Heaven could be yang changes, whilst the changes in Earth would be yin changes. If we take the lower trigram as inner and the upper as outer, this recasts Stalder’s example in terms more familiar to the Yijing. More generally, we wish to specify a particular group of lines in a hexagram where change is yang, and the complementary group of lines then have yin changes. To do this we need a technique to specify which places are potential yang changes. I introduce a hexagram I shall call the divider, D. Then we can derive X and Y as follows:

(24) $X = Q \,\&\, D$

(25) $Y = \sim(Q \,\&\, \sim D)$

From this we need to note that by combining definition (14) with equations (24) and (25) we get to the following equation:

(26) $Q = (Q \,\&\, D) \times (Q \,\&\, \sim D)$

This is a theorem of Boolean algebra, so we know that the combined definition given by (24) and (25) is consistent with bipolarity (14). Note that adopting this definition results in the imposition of a complete and disjoint division onto Q to generate the interpretation of X and Y. In turn, this automatically ensures that the minimality constraint holds for the resulting pair, because from (20) and (24) we get:

(27) $Q \,\&\, D \leq Q$
$\quad\quad X \leq Q$

As (3) above shows, the first line of (27) is a truth of Boolean algebra, so by substitution from definition (24), the second line is also true.

Now, suppose that we wish to stipulate that the changes which happen in Heaven are characterized as yang changes and changes which happen in Earth are yin changes. Then we would have:

(28) $D =$ ䷋

Which would, in turn, give the following derivations for X and Y in our example:

(29) *X* = ䷟ & ䷋
= ䷏

(30) *Y* = ~(䷟ & ~䷋)
= ~(䷟ & ䷊)
= ~䷭
= ䷘

Thus, from *X* we see that the change in the fourth line is a yang change, whilst from *Y* we see that the changes in the second and third lines are yin changes. This gives us the second pair in the list of possible pairs in (23) above, showing by example that this technique gives a form of minimal change, obeying (20).

In fact, this method, using *D*, gives a way of identifying any particular instance in the set of possible minimal pairs. Each value of *D* will give a particular *X*/*Y* pair. However, each *X*/*Y* pair is selected by a number of possible values of *D*. For example, in the context of (13), *D* = ䷲ would also select the second pair from (23).

Lattice Direction

Now we'll consider differentiating the changes in terms of their direction through the Boolean lattice of hexagrams: in this case, we must pick values for *X* and *Y* such that *X* identifies the changes moving up the lattice and *Y* identifies the changes moving down the lattice. Figure 2 shows the lattice that results for equation (13):

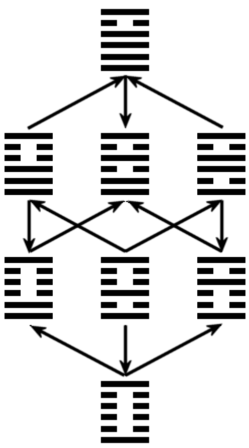

Figure 2: The Lattice for 769687

The arrows show the directions of change, flowing from *P* = ䷕ in the middle of the second layer, to *R* = ䷥ in the middle of the third layer. Changes in the third line of the hexagram result in movement down the lattice, and changes in second and fourth lines result in movement up the lattice. We can describe this flow of change by making the following general assignment to *D*:

(31) *D* = *R*

We can use *R* as *D* in this way because, if a line is yang in *R* and has changed from *P*, then it must be a movement up the lattice, whilst if a line is yin in *R* and has changed from *P*, then it must be a movement down the lattice. So, combining (24) and (25) with (31) we arrive at the following definitions for *X* and *Y*:

(32) $X = Q \,\&\, R$

(33) $Y = {\sim}(Q \,\&\, {\sim}R)$

Returning to the example developed from (13), we get the following derivations for *X* and *Y*:

(34) *X* = ䷟ & ䷥
= ䷧

(35) *Y* = ~(䷟ & ~䷥)
= ~(䷟ & ䷦)
= ~䷗
= ䷉

Thus, *X* and *Y* confirm our assertion about the direction of change through the lattice. This identifies the sixth pair listed in (23) above, and thus we see that lattice change is also a form of minimal bipolar change, obeying (20).

Change in Correctness

Correctness shows the appropriateness of yin or yang depending on their position in the symbol. We have seen that odd numbered lines are correct when yang, and even numbered lines are correct when yin, so the maximally correct hexagram is ䷾.

We can analyse changes in correctness in the same way as direction through the lattice, by picking an appropriate value for *D*. In this case, rather than using *R* directly, we use the correctness function c(.) of *R*. The result of the function c(*R*) is a hexagram where yang lines show correctness in *R* and yin lines show incorrectness in *R*. So, when we set:

(36) $D = \mathrm{c}(R)$

The yin of the change is change which makes a line incorrect, and the yang of the change is change which makes a line correct as required. Continuing with the example change in (13), we know:

(37) c(*R*) = ䷗

I'll omit the bulk of the detailed working for this example, as it follows the same formal development as the lattice change. The end result is that:

(38) *X* = ䷁

(39) *Y* = ䷩

This shows that there is no yang change, and all of the change in this example results in incorrect lines in *R*, which is the first pair in list (23). Again, we see that change in correctness is another form of minimal bipolar change.

Resonant Change

To recap, the parallel lines of the upper and lower trigrams of a hexagram are said to correspond; the first and fourth lines of the hexagram correspond, the second and third lines correspond, and the third and sixth lines correspond. When a pair of corresponding lines have different polarities, they are in resonance. When they have the same polarity, there is no resonance. Given the formal definition of resonance in (10), we can apply it as a selector of change type. As previously noted, this is more complex than our earlier examples: a line that changes may result in an increase in resonance from *P* to *R*, or a decrease in resonance; alternatively it may not result in any change in resonance, if its corresponding line in the other trigram also changes.

Because we have three possible outcomes we need to expand a level deeper into the composition of *Q*. We must begin by identifying an initial value for *D*. Based on the assertion that where there is resonance in the change quanta *Q*, then there will be a change in the resonance from *P* to *R*, we can make the following assignment:[9]

(40) $D = \mathrm{r}(Q)$

Putting this value for *D* into (24) and (25) will give us an initial decomposition of *Q* into an *X* which shows where resonance is changing and a *Y* which shows where resonance is constant:

(41) $X = Q \mathbin{\&} \mathrm{r}(Q)$

(42) $Y = {\sim}(Q \mathbin{\&} {\sim}\mathrm{r}(Q))$

However, we also wish to know, within *X*, where resonance is increasing and where it is decreasing. To do this we must apply definition (14) recursively to *X* breaking it into yang and yin aspects:

(43) $X = X^{+} \times {\sim}X^{-}$

Combining (43) with the basic definition of bipolar change (14) would give:

(44) $Q = (X^{+} \times {\sim}X^{-}) \times {\sim}Y$

This gives us three components to the analysis of resonant change: X^{+} where resonance changes and increases, X^{-} where resonance changes and decreases, and *Y* where lines change, but resonance does not change. How can we determine the appropriate values for these variables?

Following the form of reasoning applied in previous examples, if resonance has changed in a line and that line is resonant in *R*, then resonance has increased from *P* to *R*, and the change is yang. Conversely, if a line whose resonance has changed is not resonant in *R*, then resonance has decreased, and the change is yin. This means that we can use r(*R*) as the recursive selector on *X*.

(45) $X^{+} = X \mathbin{\&} \mathrm{r}(R)$

[9] A proof of this statement is omitted here, but it follows from the equivalence: $\mathrm{r}(A) \times \mathrm{r}(B) = \mathrm{r}(A \times B)$.

(46) X^{-} = ~(X & ~r(R))

Let’s now apply this recursive decomposition to the example given above in (13). Because r(Q) = ䷀ we know that all of the changing lines result in a change of resonance and, applying this value to (41) and (42), we get the following equations, confirming this:

(47) X = ䷟ & ䷀
= ䷟

(48) Y = ~(䷟ & ~䷀)
= ䷀

So, X = Q and Y = ䷀, which means that all of the change is yang, and results in a change in resonance. This is the final pair in list (23). But now we want to look inside X to see where resonance is increasing and where it is decreasing. We know that X = ䷟ and r(R) = ䷷ so putting those values into (45) and (46) we get the following equations:

(49) X^{+} = ䷟ & ䷷
= ䷽

(50) X^{-} = ~(䷟ & ~䷷)
= ䷍

So, by looking a level deeper into the structure of the change, we can now see that the changes in the second and third lines of example (13) result in an increase in resonance, whilst the change in the fourth line results in a decrease in resonance.

Further Considerations

We have seen how we can look inside change and consider its energy in terms of the interrelated yin and yang aspects of a single transformation. I have given formal definitions and presented some examples to show those definitions in action. Those examples were largely “data-driven” in that, for each case, a particular property was identified which could change when lines themselves changed, and formal definitions given which describe how changes in the lines generate changes in the given property. In the following section, I shall push the investigation further by considering the most general cases possible.

The Exhaustive Expansion of Change

Let’s begin by considering Shao Yung’s recursive development of the trigrams (see Figure 3 below, which reproduces a chart given on page 237 of Birdwhistell, 1989). This shows how a single line can be used to represent one level of change, giving two possible states; a pair of lines can represent two levels of change, giving four possible states; and three lines can represent three levels of change, giving eight possible states. The important aspect of this technique in the current context is that at each branch point, the subsequent expansion proceeds by adding a yin aspect and a yang aspect to the evolving representation.

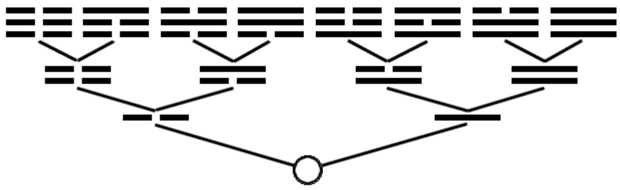

Figure 3: Shao Yung's Recursive Trigram Development

We can apply the same technique to map the recursive decomposition of change explored in this paper. Let's reconsider the example of resonant change. The first level, a single level of change, is the basic division into Q and ~Q, into change (yang) and non-change (yin). From there, we considered how Q could be expanded into transformations where resonance changed with r(Q) (yang) and transformation where resonance remained constant in ~r(Q) (yin). Then, changing resonance was further expanding into increasing resonance in r(R) (yang) and decreasing resonance in ~r(R) (yin). To highlight that this is only a partial expansion of the possible changes consider Figure 4; this shows that the expansion used in the example only recurses fully down two branches of the structure (the values used at each expansion are shown as labels).

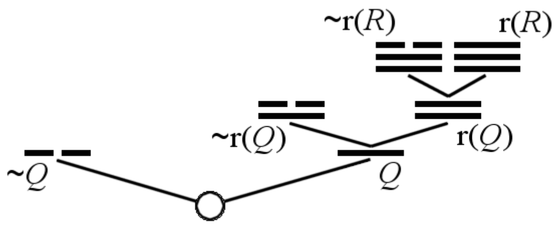

Figure 4: A Partial Expansion of Resonant Change

In this case, the partial branch for line change where resonance is stable, which terminated at ~r(Q), could be expanded another level by adding terms for stable resonance and stable non-resonance. This would complete the recursion from Q and would give us the following generalization of equation (14):

(51) $Q = (X^{+} \times \sim X^{-}) \times \sim(Y^{+} \times \sim Y^{-})$

Here X^{+} and $\sim X^{-}$ are the terms for changing resonance, as given in equation (43), and the terms Y^{+} and $\sim Y^{-}$ are the corresponding terms for stable resonance. These considerations form a natural generalization of definition (14) and, of course, the recursive expansion can be continued to an arbitrary depth. However, there is a natural point beyond which the extra categories generated have a diminishing population. Consider, if we complete the full bipolar expansion of change and non-change to three levels, then we will have eight categories of change. However, for hexagrams, there are only six lines, so typically many of the categories will be empty. Thus, although we could implement the complete decomposition, in practice one might chose to focus on particular areas of interest. In this case, we might identify some additional catagories with the following expressions:

(52) Unchanging lines where resonance increases (yang): ~Q & r(Q) & r(R)

(53) Unchanging lines where resonance decreases (yin): ~(~Q & r(Q) & ~r(R))

Thus, we see that the technique of bipolar change gives us a general mechanism for identifying detailed characteristic patterns of change and non-change within the overall energy of a transformation, and provides us with a formal language for expressing those patterns. In this case, it has allowed us to identify the case where although the polarity of a particular line does not change between *P* and *R*, a property of the line nonetheless does change as the result of other line changes in the symbol.

Dynamic Non-Change

All of the forms of change discussed above naturally conform to the minimality constraint given above as equations (20) and (21). If we look outside that constraint, an interesting possibility emerges: *X* and *Y* can be allowed to specify changes that are not in *Q*. This is possible so long as wherever *X* specifies such a yang change, *Y* specifies a parallel yin change. Reusing the example begun in (13), we can state *Q* as before:

(22) ䷟ = *X* x ~*Y*

But now consider the following equation:

(54) ䷟ = ䷳ x ~䷍

What this shows is yang change in the second, fourth and sixth lines and yin change in the third and sixth lines. But there is no change in the sixth place in the original equation:

(13) ䷕ x ䷟ = ䷙

As (54) shows, once minimality is ignored, this becomes possible because there is a simultaneous yang change and yin change, both in the sixth place; and when *X* and *Y* both have a change in the same place, there is no energy passed up to *Q*, so no difference is made to *R*. I shall call this phenomenon *dynamic non-change*.[10]

The idea of dynamic non-change is conceptually similar to the idea of virtual particle pairs and vacuum polarization in quantum physics (Penrose 2005, p676). Prior to the development of quantum theory, physicists had assumed that the vacuum between particles was a passive, empty space. However, quantum theory allows for the possibility that a pair composed of a particle and its anti-particle may come into existence for a miniscule amount of time and then vanish again, in mutual destruction. In order to avoid violating the principle of the conservation of energy, the pair of particles must cancel each other out. Further, the length of time that they can exist must be less than the time period allowed for by Heisenberg's Uncertainty Principle: the more energy there is in the virtual particle pair, the shorter their life must be. As a result, the existence of such pairs cannot be measured directly; they are disconnected from observable reality. However, if the virtual pair is charged, for example an electron and a positron, and they occur within an ambient electromagnetic field, for

[10] Note that dynamic non-change is only possible if the xor x operator is the basis of the definition of bipolar change. In Note 7 we observed that union v is also a potential basis for the definition; however, such a definition would not facilitate dynamic changelessness. As Note 7 provided another reason to prefer the use of xor, dynamic non-change remains a possibility for exploration.

example near a “real” electron, then the pair separate slightly, and cause any measurement of the ambient electromagnetic field to be weaker than it should be. This is the effect referred to as vacuum polarization.

The parallel with dynamic non-change is clear: when dynamic non-change happens all throughout Q there is no change manifest. Note that this situation is allowed for in (16) above. However, when dynamic non-change happens in the context of some definite non-ground value for Q, then it serves to dilute the changes individually observable in X and Y. To see this, consider the example minimal values for X and Y given for lattice change in (34) and (35):

(55) Q = ䷧ x ~䷈

Now compare this to the values given in (54) above, shown here as separate equations, using a subscripted d to indicate the values of X and Y contain dynamic aspects:

(56) X_d = ䷿

(57) Y_d = ䷹

So, with dynamic non-change we see the following two inequalities:

(58) $X < X_d$

(59) $Y_d < Y$

Meaning that, where X and Y show the actual patterns of yang and yin change in Q, X_d shows more yang change than X and Y_d shows more yin change than Y. Thus, dynamic non-change dilutes the information carried in the individual components of change and we cannot be sure where, in Q, the yang changes are merely by looking at X, we must also look at Y. Similarly, of course, the yin changes no longer depend purely on Y, but also on the content of X.

The question then arises as to the maximum amount of dynamic non-change that can occur within any given non-ground Q for a particular, initial minimal division into yin and yang. That is, given some decomposition of Q into an X and Y obeying the minimality constraint, what is the most change in hexagrams X_d and Y_d (let’s call them X_{max} and Y_{max}) such that:

(60) $Q = X_{max}$ x $\sim Y_{max}$

(61) $X \leq X_{max}$

(62) $Y_{max} \leq Y$

We can describe X_{max} and Y_{max} verbally: X_{max} must have all of the yang change in X plus yang change wherever Y does not have yin change; similarly Y_{max} must have all of the yin change in Y plus yin change wherever X does not have yang change. From these two descriptions we can use the basic definitions of Boolean algebra to write down the required logical definitions of the two terms (recall the explanations given for the & and ∨ operators based on Table 1):

(63) $X_{max} = X \vee Y$

(64) $Y_{max} = X \,\&\, Y$

The solution to these constraints is an initially surprising pair:[11]

(65) $X_{max} = Y$

(66) $Y_{max} = X$

What can we make of this definition of maximality? For any particular minimal change *X* and *Y*, the maximal consistent dynamic non-change in yang has the same representation as the minimal yin change. Conversely, the maximal consistent dynamic non-change in yin has the same representation as the minimal yang change. Or, put more simply, when yang reaches its maximum, it becomes yin and, when yin reaches its maximum, it becomes yang.

Conclusions

In this paper I have explored how an absolute notion of change, encoded in the natural algebraic definition, can be relativized to allow us to explore its internal structure, comparing different aspects of a change in terms of their yin and yang nature. Beginning with the standard algebraic definition of change, which represents the energy of change by a quanta hexagram *Q*, we considered how this could be broken into a yang component *X* and a yin component *Y*. This technique facilitates the formal representation of change in terms of its direction of movement through the Boolean lattice, in terms of transformations to the patterns of correctness, and in terms of transformations to the patterns of resonance. In exploring resonant change, we also saw that it was possible to expand the internal structure of bipolar change as many times as were needed to extract the required information.

Each of these particular instances of bipolar change were derived from the more general technique of dividing the changes into two groups and designating one as yang and the other as yin. Further, all of these instances of bipolar change were minimal, conforming to a natural constraint on their information content. Drawing a conceptual parallel with vacuum polarization in quantum physics, we saw that there are other forms of bipolar change, embodying dynamic aspects which, nonetheless, do not result in manifest change. I suggest that this work therefore extends the standard algebraic definition of change, placing it in a broader context, and revealing new aspects which were previously hidden.

However, there remain classes of bipolar change whose constraints cannot easily be expressed mathematically. The end result of the change can be represented using the techniques described above, but these results would be interpreted directly from individual experience and then projected onto the symbols. For example, there is no obvious mathematical description which distinguishes changes which arise from the intrinsic energy of the situation from those which arise as a result of an extrinsic impetus to change. Unless a convincing formal description can be constructed, such a

[11] In fact, (65) and (66) follow from the definitions (63) and (64), via (14) and (20), once the following theorem of Boolean algebra is applied: if $Q = X \times {\sim}Y$ and $X \leq Q$ then $X \leq Y$

categorization must remain an act of personal interpretation based on perception of the actual situation. In such a case, the analyst would have to consider the details of the situation and then decide on an appropriate value of D to use in equations (24) and (25). Only once that decision has been made could the formal analysis proceed.

In this connection, we should also reconsider Wilhelm (1983) and Cheng (2005), which both provide classifications of the possible forms of change. At the base, for both, is non-change, $\sim Q$ in the notation used here. Beyond that, both authors divide change into different categories. For Wilhelm, change is either linear or cyclic. Can we describe this difference using the mechanism explored in this paper? In this case, the answer is "not mechanically" – because we are only dealing with a single change from one symbol P to another R, it is not possible to decide whether the eventual outcome will be linear or cyclic. Thus, we have no formal, algebraic basis for distinguishing these forms of change and, as outlined above, the decision for D has to rest with the person performing the analysis.

In Cheng's work, change has four categories: i) change, as becoming different; ii) simplicity, as an underlying principle in all transformation; iii) exchange, where two or more system exchange energy; iv) and harmonization, which provides an over-arching principle that guides all of the activity of change. In this case, two of the categories are described as guiding principles which apply to all instances of change. Therefore, it is not a case of deciding whether a change has simplicity or harmonization: both should always be active. Therefore, these two categories do not fit directly within the bipolar model of change presented here. This leaves change and exchange to consider. As noted, in the first case, we could say that change is a single system becoming different, whilst exchange involves two systems. Deciding which aspects of the transformation are basic change and which are exchange will depend on an understanding of the situation that the symbolic change is describing. Again, this clearly depends on context outwith the formal algebraic properties of the symbolic language and, as such, we cannot provide an algebraic formulation to distinguish them and it would be down to the analyst to identify an appropriate value for D.

So, although the extended symbolic mechanism of bipolar change has opened up a wide range of new possibilities for describing the formal properties of change, it is not without its limitations. In order to be applied effectively, it must be possible to provide a formal, algebraic characterisation of the properties under consideration. The extended examples involving lattice direction, correctness and correspondence all have this property. Many other aspects of change do not, and will therefore rely of the sensitivity of the analyst if bipolar change is to be applied.

References

Birdwhistell, Anne D. 1989.
Transition to Neo-Confucianism: Shao Yung on Knowledge and Symbols of Reality. Published by Stanford University Press, Stanford California.

Boole, George. 1854.
An Investigation of the Laws of Thought on which are Founded the Mathematical Theories of Logic and Probabilities. Originally published in the US by Macmillan in 1854, republished in 1958, by Dover, New York.

Cheng, Chung-Ying. 2005.
“On Five Senses of Yi (Change/Creativity) and the Onto-Cosmological World of Yi” in *Proceedings of the 2nd International Conference on I-Ching (Yijing) Studies and Contemporary Civilization*, pp17–35.

de Fancourt, William. 1997.
Warp and Weft: In Search of the I-Ching. Capall Bann Publishing.

Goldenberg, Daniel S. 1975.
“The Algebra of the I Ching and its Philosophical Implications” in *Journal of Chinese Philosophy*, Volume 2, Number 2, 1975, pp149–179.

Karcher, Stephen. 1997.
How to Use the I Ching: a Guide to Working with the Oracle of Change. Element Books, Shaftsbury, Dorset.

———. 2003.
Total I Ching: Myths for Change. Time Warner Books, London.

Karcher, Stephen and Andreas Schöter. 2002.
“Tools for Change” in *The Oracle: the Journal of Yijing Studies,* Volume 2, Number 12, January 2002, pp20–33.

Nielsen, Bent. 2003.
A Companion to Yi Jing Numerology and Cosmology:Chinese Studies of Images and Numbers from Han (202 BCE – 220 CE) to Song (960 – 1279 CE). Published by RoutledgeCurzon, London.

Penrose, Roger. 2005.
The Road to Reality: A Complete Guide to the Laws of the Universe. Published by Vintage Books, London.

Schöter, Andreas. 1998.
“Boolean Algebra and the Yijing” in *The Oracle: the Journal of Yijing Studies*, Volume 2, Number 7, Summer 1998, pp19–34.

———. 1999.
“Correctness and Correspondence” in *The Oracle: the Journal of Yijing Studies*, Volume 2, Number 8, February 1999, pp27–35.

———. 2004.
“Flowers and Steps in the Boolean Lattice of Hexagrams” in *Journal of Chinese Philosophy*, Volume 31, Number 4, December 2004, pp489–504.

———. 2005.
“The Yijing as a Symbolic Language for Abstraction” in *Proceedings of the 2nd International Conference on I-Ching (Yijing) Studies and Contemporary Civilization*, pp291–305.

Smith, Richard J. 1998.
“The Languages of the Yi Jing and the Representation of Reality” in *The Oracle: the Journey of Yijing Studies*, Volume 2, Number 7, Summer 1998, pp35–50.

Stalder, Alain. 2005.
Exact Philosophy. A web site at http://www.exactphilosophy.net

Wilhelm, Richard. 1983.
I Ching or Book of Changes. Rendered into English by Cary F Baynes. Published by Routledge & Kegan Paul, London.