

The Yijing as a Symbolic Language for Abstraction

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Introduction

The symbols of the Book of Change present an astonishing enigma. Their origins lie deep in China's past, born from the divinatory practices of the Zhou dynasty – but the patterns of open and closed lines resonate down through the millennia to the present day. This paper looks at the symbolic language of the Book of Change, applying techniques from Boolean algebra to generate and analyze structures which represent the relationships between the symbols.¹ This process is part of an intellectual lineage that began well before the Yijing had contact with the West. Shao Yung's work on the binary sequence of the symbols² and the various diagrams in the *Zhouyi Tuishi Dadian* make this clear.^{3,4} More recently this thread was continued in Z. D. Sung's geometric characterizations from the 1930s.⁵ The first modern mathematical work on the Yijing in the West that I am aware of is by Goldenberg in 1975 who bases his approach on modulo arithmetic.⁶ In contrast, the algebra used here is comes directly from computer science and integrates with the nature of the symbols of the book.

The main idea that this paper presents is a mathematical way of representing multiple perspectives on reality. The whole of reality is too complex and detailed to be grasped as one. Instead, each of us sees a partial view from our own perspective. By starting with the idea that the symbols of the Yijing encompass the totality of reality, we shall see that by focusing awareness on different dimensions of experience, by concentrating on different abstractions, different perspectives naturally unfold. This idea is given a precise formal presentation, by showing how the Yijing can be seen as a symbolic language, and then exploring the algebraic properties of that language.

The intention here is to introduce readers versed in the traditional study of Change to a contemporary view of the book that is, nonetheless, deeply rooted in the tradition of the Image and Number school. To this end, modern mathematical techniques are applied to analyse the structures of the trigrams and hexagrams. The philosophical implications of the formalism are discussed and the structures are then interpreted in a cosmological context, using ideas and language borrowed from quantum physics and logic, to provide rich metaphors for describing the structure of reality.

Abstraction, Notation, and Representation

Before diving into the mathematics of the matter we should first explore what is meant, in this context, by the terms *abstraction* and *symbolic language* and how they are applied to the Yijing.

Abstraction

The ability to abstract common features from diverse situations is a key cognitive function, vital in the formation of concepts and our day-to-day functioning. When we look at a ball, a hoop, a wheel, and the full moon we might recognize something that they have in common: namely, roundness. But what is “roundness”? We cannot point to any particular *thing* in the world and say “that is roundness”; we can only point to particular objects and say “that thing has the property of roundness”. Roundness is a concept that we abstract as a property from a wide range of concrete objects which exhibit it.

Seeing what is common in things that appear different enables us to generalize about situations and apply techniques learnt in one set of circumstances to problems encountered in another. We can understand the symbols of the Yijing as representing abstract concepts that can be applied to novel situations as they are encountered. The symbols give us a set of categories that we can use to organize and interpret our experience.

Binary Representation

In today's digital world, bit-wise representation has become the dominant means of recording and processing information. The word "bit" is a contraction of the phrase "binary digit" and a bit, which entokens the distinction between 0 and 1, between nothing and something, is the smallest possible unit of information. In physics, experience shows that the status of any particular "particle" as a basic building block of reality is contingent on further experimentation; indeed, the idea of a particle itself becomes questionable. However, in information theory there is no case for supposing that this binary distinction can be further reduced. Although there might be uncertainty over the value in any particular instance, the distinction between 0 and 1 is to be taken as conceptually fundamental.

Binary representation provides the basic formalism for computer programs and their rich algorithmic possibilities. It is the very fundamentality of the binary notation that provides such tremendous representational versatility.

Symbolic Language

What do we mean by a *symbolic language*? In the context of this discussion, we shall take it to be composed of some fixed set of symbols (the words of the language), and some rules for transforming between symbols and for combining groups of symbols into structures (the grammar of the language). The symbols themselves are taken to have meaning, where this meaning comes about by virtue of reference to objects in the domain being symbolized. The rules themselves should also have meaning, representing processes and structures in the domain.

Enumerating all the possible combinations of open and closed lines over six places, the 64 hexagrams can be seen as the first systematic, symbolic language for categorization. The hexagrams themselves are the symbols of the language. The traditional method of representing changing lines, which generates relationships between pairs of symbols, is one example of a rule of symbol transformation. Other traditional elements of the Yijing which provide the transformation rules include the extraction of nuclear hexagrams, and the various methods of deriving opposite hexagrams.⁷

The traditional domain of application for the notation in the Yijing could be described as the study of the interaction between human consciousness and its environment. That is, the hexagrams provide us with a binary notation for describing the interactions of the various forces that are at work in the universe, and our place relative to those forces. The narratives attached to the symbols record the traditional interpretations of those interactions, but the trigrams and hexagrams themselves are also taken to provide a direct, iconic image of the forces at work.

Mathematics

The roots of binary encoding, in the symbols of the Yijing, go back many millennia in China. However, in the West it begins much later. Leibniz, working in the second half of the 17th century, was concerned with exploring arithmetic using binary representation. Later in

his life, after coming in to contact with the ideas of the Yijing by way of the Jesuit missionary Bouvet,⁸ his understanding of binary numbers became very mystical, believing that they represented Creation. He saw the number 1 as representing God, and 0 as representing Void. In the traditional language of the Yijing, the terms Creative and Receptive could be substituted.

Mathematics is a language of universal truth, and as a result mathematical analysis has a great deal to contribute to the study of the Yijing. It allows us to make explicit some of the structures and relationships between the symbols that would otherwise not be visible. Once revealed, those structures then provide us with a rich source of metaphor to explore and apply. For those readers who do not have any mathematical background, I hope that the inclusion of numerous tables and diagrams will help to make the concepts clear.

Interpreting the Gua

The gua provide us with the words of our symbolic language, and it is worth while making explicit some assumptions about their role and interpretation in a general system of abstraction. We shall begin by looking at two contemporary ideas, one from logic and one from physics, and show how their combination gives us a picture of meaning for hexagrams.

The Logic of Situations

There has been a shift of emphasis in formal logic in recent decades. When logical calculi were first conceived, the idea of a proposition was an isolated atomic entity that stood alone and whose truth could be evaluated in a manner that was independent of the context of its use. This formulation has now given way to a richer, more contextual notion. The idea of a *situation* has come to the fore in some circles, where entities and the relationships between them provide the context for evaluating statements. Barwise and Perry were the first to make formal use of situations. They say:⁹

Reality consists of situations – individuals having properties and standing in relations at various spatiotemporal locations. We are always in situations; we can see them, cause them to come about, and have attitudes towards them.

Following the development of this position in formal semantics, each hexagram can be seen as representing an abstract situation.¹⁰ As an abstract situation, each hexagram encompasses a group of actual situations, forming a category that can be applied to a range experiences, helping to make sense of those situations by drawing out common themes and relationships. In this paper, the meaning of the gua can be constructed by generalizing certain key properties of situations in terms of the energetic interactions within the situation, where those interactions are described by relating the different components of the whole to each other. How a hexagram is internally divided and the energetic interactions that arise can be described by making use of a recent idea from physics.

Quantum Physics

In response to a number of problems with the theory of quantum mechanics, the physicist David Bohm developed a theory explaining physical causality and the structure of reality using two domains which he called the explicate order and the implicate order.¹¹ The explicate order is the everyday reality which is revealed to our senses and our measuring instruments. In contrast, the implicate order is the deep, underlying aspect of reality which “unfolds” to give expression to the explicate order. Events that happen in the manifest,

explicate world happen because of the patterns in the implicate order. Thus, physical causality in this world, the explicate order, becomes a secondary phenomenon, subordinate to the unfolding activity of the implicate order. This bears a striking similarity to the relation between Heaven and Earth in the Yijing: the implicate order is Heaven and the explicate order is Earth. Activity in the former determines events in the latter.

Bohm further suggests that consciousness arises as an interaction between the implicate and the explicate orders. This happens because, just as the implicate order unfolds to give rise to the explicate reality, so our mind enfolds the explicate order back into the implicate order of conscious memory. He uses our perception of music and melody as an example. As we hear each note, it is not experienced in isolation, but in terms of the context of all the notes that have come before it in the piece. As each new note is heard, it is enfolded into the ongoing memory of the music, building up a mental structure which creates expectations reaching forward into the future. The implicate order drives the reverse of this process: each moment of explicate reality is enfolded within the implicate order, becoming manifest as it is unfolded.

The parallel with the Yijing is clear: just as consciousness emerges as the interaction between the implicate and explicate orders, so man arises through the interaction of Heaven and Earth. There is also a striking parallel between the continual unfolding and enfolding of the implicate and explicate orders and a text from the *Shuo Gua* which says:¹² “The numbering of the past is flowing with the current. The knowledge of the future is countercurrent.”

The Internal Structure of Hexagrams

By combining the ideas discussed above, it is possible to describe the internal structure of hexagrams using a different language, which is nonetheless in accord with the cosmology of the Yijing. We start with the idea of a hexagram representing an abstract situation, with a range of contributing energies arranged from most explicate in the bottom line to most implicate in the top line. Then we add the standard division into two primary trigrams, as shown on the left of the table below; following Bohm, these represent the explicate and the implicate order. Consciousness arises through the interaction of these two domains, creating a third realm unfolding from their intersection, to give the three bigrams shown on the right on the table.

Implicate	{	Pattern Spirit Mind	}	Implicate
		Heart	}	Consciousness
Explicate	{	Body Matter	}	Explicate

Table 1: The Primary Internal Structure of the Hexagram

This leads to the following attribution of individual lines for a situation: at the most explicate end of the spectrum is matter, the concrete stuff that allows patterns to become manifest; then, the body is the level at which actual things are individuated, the initial separation into discrete beings. The broad categories of process that build and sustain consciousness are next: most deeply meshed into the biochemical process of the physical body is the emotional response, the heart; more abstracted is the mental intellectual level, where we first begin to apprehend the implicate order. The fifth layer defines the nature of what is actually enfolded; as the

implicate equivalent of the explicate body, it is what connects the manifest to the purely abstract, this is spirit. Finally, at the most implicate end of the spectrum is pattern, pure abstract structure without specific content.

It is interesting to note that, starting with the broad categories from physics and proceeding by a process of refinement, we have arrived at an assignment of meaning to lines which is very similar to that suggested by Marysol Gonzales Sterling who approaches the matter from the direction of transpersonal psychology.¹³ Whilst different in detail, this characterization of the places of the hexagram is similar in method and spirit to the attributes put forward by Thomas Cleary in his book *I Ching Mandalas*.¹⁴ He discusses a number of different systems for assigning meaning to the places based on Daoist, Buddhist and Confucian ideas. These use a variety of social and psychological hierarchies as their basis, rather than the physical abstraction employed here. However, the method of dividing reality into repeatedly finer domains is a technique common to many of the characterizations.

Another classically significant division of the hexagram is into the two overlapping nuclear trigrams. We shall not consider this in detail here, but each way of dividing the hexagram gives a different way of classifying the energies involved by focussing the attention onto a specific subspace. We shall see later that these different divisions give rise to different structural unfoldings of the totality, where the meanings of the unfoldings can be derived from the internal structure of the hexagram outlined here.

A Graphical Introduction to Lattices

An algebraic system consists of a set of symbols (the gua) and a collection of operators that act on those symbols to combine and transform them. In turn, those operators generate a set of structures that organize and classify the gua; these structures are lattices.¹⁵

One-Dimensional Lattices

There is a very natural connection between the complexity of the structures generated by a Boolean algebra applied to the gua and the geometric idea of *dimensionality*. In what follows we shall start with single line structures and build up to hexagrams. So, for single lines, the lattice shown in Figure 1(a) below is generated:

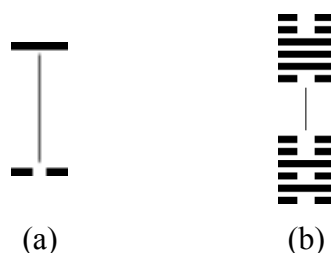


Figure 1: One-Dimensional Lattices

This shows the two possible states that a single line might be in and the energetic relationship between those two states. Thus, a line might be either yin or yang, and a yang line has more energy than a yin line. In terms of information theory, we would say that this represents the two possible situations that a single binary category can distinguish, it is the basic polarization. In geometric terms, this is a one-dimensional structure, or an *edge* in more complex structures. For example, the one-dimensional lattice shown as (b) above connects the hexagrams Duration and Deliverance¹⁶ with the lattice edge being formed by the energetic

difference in the third line. Note that there are 192 edges in the complete hexagram lattice, which encode the 384 single line changes of the Yijing.¹⁷

Two-Dimensional Lattices

When we consider pairs of lines, the lattice shown in Figure 2(a) below is generated. This shows the energetic relationships between the four possible states that can arise from two lines. At the bottom, analogous to Receptive, is the pure yin bigram with the lowest energy level; and at the top is pure yang, Creative with the maximum energy level. On the left, analogous to Thunder is a yang line bursting forth beneath yin, beginning the cycle around the lattice. On the right, analogous to Mountain, is a yang line resting above yin, ending the cycle around the lattice. When moving between any two adjacent gua in the structure, only a single line of energy changes. Thus, to borrow a term from physics, each edge of the lattice describes a minimal quantum state change between two situations. Geometrically, this lattice for bigrams is two-dimensional and we will see that, projected up to hexagrams, it generates facets of the whole lattice.

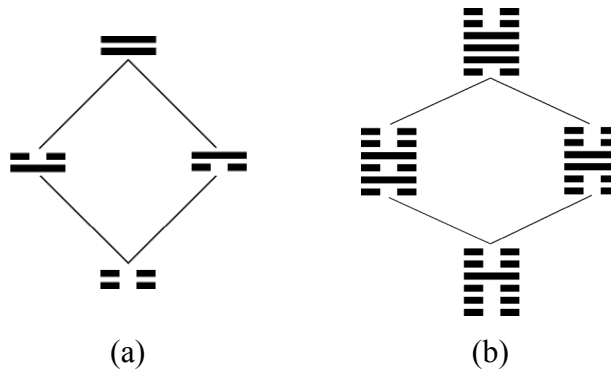


Figure 2: Two-Dimensional Lattices

Consider the two-dimensional structure shown as Figure 2(b) above, this is created as the result of the energetic differences in the second and third lines of the hexagrams: notice that the edge between Duration and Deliverance shown in Figure 1(b) above appears here as one of the four edges of the facet. Note that there are 240 unique facets in the complete hexagram lattice, encoding the 960 changes involving two lines. We will see next how this facet itself appears as part of a cube.

Three-Dimensional Lattices

Finally in this section, we consider the cubical lattice for trigrams shown in Figure 3(a) below. As before, an interpretation of the structure in terms of energy state is natural. Receptive is at the bottom, with no yang energy. In contrast, Creative is at the top, with maximum yang energy. The other six trigrams are arranged in layers between, with first one line of yang energy, and then two. The lattice represents the various states of the situations, defining the field of quantum potential for trigrams, and the connections within the lattice show the minimal transitions from one state to another, the switching of a single line.

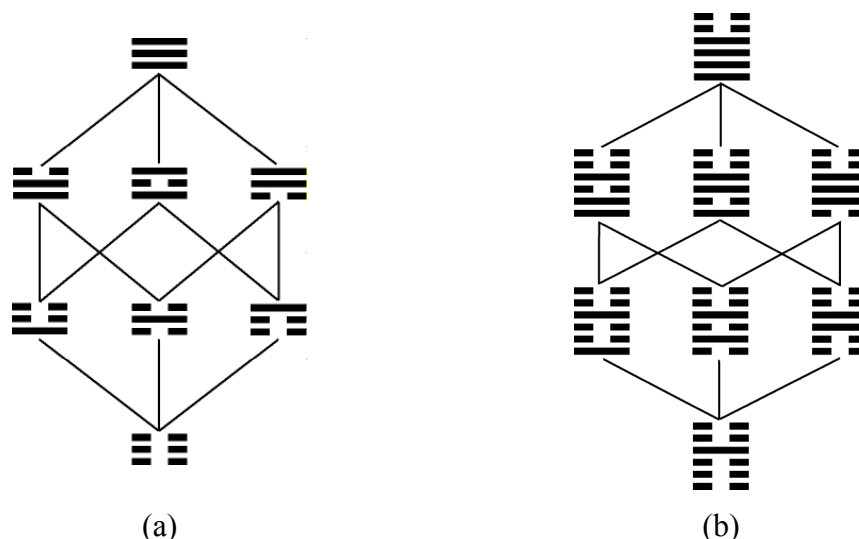


Figure 3: Three-Dimensional Lattices

Geometrically, the trigram lattice is a three-dimensional structure, which each trigram connected to others through the changes of its lines. Now consider a corresponding three-dimensional figure for hexagrams shown as Figure 3(b). We see a cube of hexagrams, formed as the result of the energetic permutations of the lower trigram. The hexagram Power of the Great is at the top and Enthusiasm is at the bottom. Notice that the two-dimensional lattice shown as Figure 2(b), between Enduring and Enthusiasm, appears as a facet of this cube. There are 160 unique cubes in the hexagram lattice encoding the 1280 changes involving 3 lines.

So, we can see that the structures created at one level of dimensionality are projected upwards into the higher dimensional structures. The lines of the one-dimensional lattice form the edges of two- and three-dimensional lattices. The squares of the two-dimensional lattice form the facets of the three-dimensional lattice. This pattern continues up through the higher dimensional structures until the six-dimensional lattice of the hexagrams is reached. This is a complex structure which is hard for our three-dimensional visual minds to grasp in its totality (see Figure 7 at the end of the paper). Later, we shall present ways of making it more comprehensible by unfolding it to different degrees. But first, we need to provide some mathematical tools for analysing these structures.

Ordering the Gua

The most important aspect of the algebraic definition of a lattice is the *partial order* that defines it. This idea is best understood by comparing a partial order to a *total order*. In making this comparison notice how the terminology changes as we switch from a total to a partial order; this is more than just verbiage, a change in meaning happens. Notice also how different properties hold for the two different ways of ordering symbols.

In both total and partial orders, we shall write $g < h$ to mean that gua g comes before h in the order under consideration. If we wish to allow for the possibility that g and h might be equal in the ordering, then we would write $g \leq h$ to say that g either comes before h or is equal to h in the order.

Total Orders

An example of a total order is the King Wen sequence of the hexagrams.¹⁸ In that case, Creative comes before Receptive, and Receptive comes before Limitation. These facts can be written as follows:



Note that the first of these orderings is immediate, Receptive comes directly after Creative, but in the second case Limitation comes some way after Receptive in the sequence. This does not matter to the notation. If we only knew these first two facts about the King Wen sequence, we would be able to deduce that Creative comes before Limitation directly from the properties of the ordering relation, thus:



This property is called *transitivity*, and is represented mathematically by the rule:

$$\text{if } f < g \text{ and } g < h, \text{ then } f < h$$

In a total order defined over a finite set of symbols, there is some symbol that starts the sequence. In the case of the King Wen sequence, this is the hexagram Creative. Let us call this the *first* symbol, because there is no symbol *before* it in the sequence. If we assume that the sequence does not loop, then such an order also has a symbol we shall call the *last* symbol, with no symbol *after* it in the sequence. In the Yijing this is the hexagram called Before Completion: to capture both of these facts, we would write:

$$\text{No } h < \text{☰} \quad \text{No } h \text{ ☱} < h$$

Now, when an order is total, either $h < g$ or $g < h$ for any two distinct gua in the set. This is clearly the case: if the sequence contains all of the hexagrams arranged in a line, then for any two hexagrams, one must come before the other. It is this property that fails when we turn to consider partial orders.

Partial Orders

Firstly, note that the property of transitivity described above holds for partial orders just as it does for total orders. We could say that transitivity is one of the defining characteristics of an order of any sort. However, in a partial order, rather than having first and last symbols, we have *bottom* and *top* symbols. In the case of the algebra presented here, the ordering is conceptualised in terms of energy levels. Therefore, the bottom symbol is Receptive, with no yang energy and there is no hexagram lower than Receptive. Similarly, there is a *top* symbol, with nothing *higher*. In the Yijing, this is the Creative, pure yang energy. Using the same notation as before:

$$\text{No } h < \text{☷} \quad \text{No } h \text{ ☰} < h$$

So, on moving to a partial order the idea of first and last symbols are replaced by the concept of bottom and top symbols. However, in the case of the lattice structure the order is not total, it is *partial*. In this case, for example Peace is not below Standstill and Standstill is not below Peace:



Taken together these inequalities say that Peace and Standstill are not ordered relative to each other. This can be written more concisely as:



These two hexagrams occur in the same layer of the lattice. Any two gua in the same lattice layer are clearly not up or down relative to each other because each is the same distance between Receptive and Creative. Further, because of the different distribution of yin and yang lines within the hexagrams, neither are they energetically equal to each other either. In additions to gua in the same layer in the lattice, there are symbols in different layers which are not ordered relative to each other.

The Algebra of the Gua

Having explored the partial order, we can now look at the algebraic operators themselves and how they interact with the ordering. Conceptually, the operators tie the links of the chains together into the lattice structures presented above. We could say that a lattice shows us how the gua delineate space by threading connected chains through it.

The Complement Operator

The first Boolean operator is the *complement* operator (sometimes called *Boolean negation*). This transforms yin to yang and yang to yin and can be represented using the table shown.

l	$\sim l$
— —	— —
— —	— —

Table 2: The Complement Operator

This simple operation reverses the polarity of the line to which it is applied. When we extend this to trigrams and hexagrams, the operator is applied to each line in turn to give the combined result. With trigrams, for example, the complement of Arousing is Penetrating:



Look back to the lattice diagrams to see the geometric picture. Interpreting the complement operator in energetic terms then this should be inverting the overall energy of the gua. This is confirmed in terms of a property of the partial ordering relation:

$$\text{if } f < g, \text{ then } \sim g < \sim f$$

Note that complementary pairs of trigrams are diagonally opposite corners in the Sung cube. Also, the Fu Xi arrangement of trigrams organizes the trigrams into complementary pairs. Finally, the complement operator is the algebraic description of the principle used in the King Wen sequence to organize the linearly symmetric pairs, such as Abysmal and Clinging. So, the complement operator is part of the standard apparatus of the Yijing.

The Lattice Operators

The meaning of the next two operators of the algebra are partly expressed by the English words “and” and “or”. These operators are used to build the lattice structure for the gua and they are crucial tools for unfolding the complete lattice into smaller substructures.

Take the “and” operation first. As before, we begin by restricting our attention to individual lines. This operator takes two lines and combines them to give a new line related to both of the original lines. The particular mode of composition for “and” is shown in the following table.¹⁹ In words: if both of the combining lines are yang, then the result of the combination is also yang, otherwise the combination is yin:

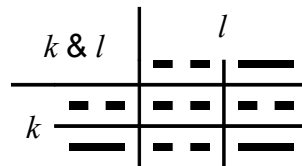


Table 3: The Lattice Divergence Operator

This applies to trigrams line by line:



So, the “and” combination of Lake and Wind is Water. Look back at the trigram lattice diagram, Figure 3(a), and notice the structural relationship between these three trigrams. Water is at the lower vertex of Lake and Wind. Thus, the “and” operator creates the downward reaching elements of the lattice structure. It is the *greatest lower bound* of the energy states of the two original gua; in terms of the partial order it is the *most* energetic state included in *both* of the combining states. We can say that it is the point immediately before the energy of the two gua diverges. If we look at the lattice, it is clear that, in its role as the bottom element of the structure, Receptive is the point of universal divergence.

The second lattice operator is “or”, and the mode of combination for this is shown in the next table. If either of the combining lines is yang, then the resulting combination is yang, otherwise the combination is yin.

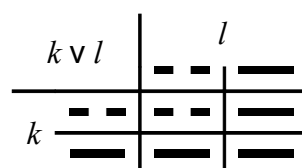


Table 4: The Lattice Convergence Operator

Take the following equation as an example of its application to trigrams:



This says that the “or” combination of Thunder and Mountain is Fire. Again, look back at the trigram lattice and notice the structural relationship between these three trigrams: Fire is the upper vertex of Thunder and Mountain. Therefore, the “or” operator creates the upward reaching elements of the lattice structure. Energetically, this is the *least upper bound* of the energy states of the two original gua; it is the *least* energetic state that includes both of the original gua; their point of convergence. Again, if we consider the role of Creative as the top most energy state in the lattice, then it can be seen as the point of universal convergence.

When we consider the result of the algebraic operators applied to the gua, geometric structures are created which can be seen to represent the basic energetic relationships between those gua. Expanding on this analogy, we say that each position in the lattice represents a particular quantum state of the situation; then, the “and” and “or” connections define points of divergence and convergence for pairs of states.

Change as Difference

I shall now describe how energy states are compared using an operator sometimes called “exclusive or”. Viewed asymmetrically, this describes how lines change as the result of an input of energy: in this role the operator provides an analysis of change.²⁰ Here I shall show how it can be used to unfold the complete hexagram lattice into manageable chunks. The table for this operator is given below:

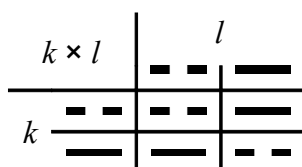


Table 5: Multiplication of Energy States

Consider an example of applying this to trigrams:



This shows that the difference between the energies of Lake and Fire is Wind. The two yang lines of Fire, in the upper and lower places cause the corresponding lines of Lake to change into their opposites.

When applying this operator to hexagrams it can be used to create a subspace of the complete lattice corresponding to the total energy potential of the differences being considered. As in previous sections, we shall start with a diagrammatic example (see Figure 4 below).

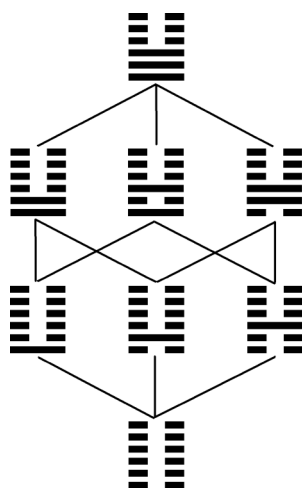


Figure 4: Example Cubic Hexagram Lattice

There are 8 equations that we could use to describe this cube. We shall consider the change from Approach to Modesty generated by Peace, represented by the following equation:



We shall say that Approach is the primary hexagram, Modesty is the resulting hexagram and Peace is the energy hexagram describing the change quanta involved. In fact, Peace is the change quanta governing the entire cube. In addition, we need to consider the following two equations, the lattice combinations of the primary and related hexagrams:

$$\begin{array}{c} \text{☰} \\ \text{☷} \end{array} \& \begin{array}{c} \text{☰} \\ \text{☷} \end{array} = \begin{array}{c} \text{☰} \\ \text{☷} \end{array} \quad \begin{array}{c} \text{☰} \\ \text{☷} \end{array} \vee \begin{array}{c} \text{☰} \\ \text{☷} \end{array} = \begin{array}{c} \text{☰} \\ \text{☷} \end{array}$$

The first says that the divergence of Approach and Modesty is Receptive, and the second says that the convergence of Approach and Modesty is Peace. The cube is then formed by taking every hexagram in the lattice that is between Peace and Receptive. The following definition describes how this is done in mathematical terms.

The sub Function

$$\text{sub}(p, q) = \{g \in L : p \& (p \times q) \leq g \leq p \vee (p \times q)\}$$

What does this function mean? Let us take this step by step, considering how each component of the definition contributes to the overall meaning. Firstly, $\text{sub}(p, q)$ is the name of the function we are describing, this is going to define a subspace of the overall lattice, based on some primary hexagram p and an energy hexagram q specifying the change quanta. In the example above, p would be Approach and q would be Peace, then $p \times q$ would be Modesty. The complex expression to the right of the equals sign tells us that the result will be a set of hexagrams g taken from the total lattice which fall between the convergence and divergence of p and $p \times q$.

Note that cubes are always formed by a sub function with a q parameter chosen from the 20 hexagrams with three yang lines. Similarly, two-dimensional facets are always formed from a sub function with a q parameter chosen from the 15 hexagrams with two yang lines. In general, if the q parameter to the sub function has n yang lines, then the result will be an n -dimensional subspace. So, the more energy there is in q , the more complex the resulting substructure it creates. The sub function gives us an easy way of creating the full range of subspaces. If we chose Receptive for q , with no changing lines, then we will get a zero-dimensional subspace, that is, the single hexagram corresponding to p ; if we chose a hexagram with one changing line, for example Return, then we will get a one-dimensional subspace and so on. If we chose Creative, then we get a six-dimensional subspace which is, of course, simply the complete lattice itself.

How Everything Unfolds

Now we have the sub function, we are able to explore how to unfold the six-dimensional hexagram lattice into a variety of simplified structures.

The Multidimensional Nature of Reality

We have seen that, following the geometric story, the Boolean algebra generates a six-dimensional lattice structure when applied to hexagrams; this is the fractal repetition of the three-dimensional trigram cube up to the higher dimensional space of hexagrams.

The idea that reality consists of more than the three-dimensional space that our senses reveal to us can be understood in many different ways. It is a conjecture explored in contemporary physics through the topic of string theory.²¹ This theory postulates that there are additional

dimensions of reality, folded up within the normal three that we are used to experiencing. Most versions of string theory postulate 10 dimensions in total; however, the exact number of dimensions is of less interest than the idea itself.²²

In the case of the Yijing, the dimensions that we are representing are not only literal physical dimensions, but also dimensions of experience, emotional and psychological dimensions, dimensions of thought and abstraction. We shall see below, by exploring different ways of unfolding the hexagram lattice, that different ways of focussing our attention result in very different perspectives on reality.

Unfolding the Complete Lattice

The purpose of unfolding the complete lattice is to break it down in to manageable substructures, where each substructure is easier to understand than the totality. Also, because conscious awareness itself is typically partial and changing, these unfoldings are a better conceptual representation of our actual experience. By using the appropriate mathematical definitions, we can define a set of substructures which, taken together, form a natural sequence that covers all of the lattice, and where each substructure within the sequence provides a coherent grouping of hexagrams, reflecting the overall structure of the lattice. That is, each unfolding provides a holographic fragmentation of the whole lattice, mirroring its structure at different degrees of dimensionality. Diagrammatic examples of complete unfoldings are shown at the end of the paper.

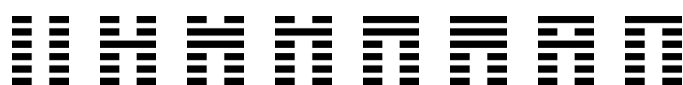
For example, we could consider unfolding the complete lattice into eight cubes, where each cube contains eight hexagrams, and all the cubes taken together cover the complete lattice, fitting together into a larger hypercube. Alternatively, we could unfold the lattice into 16 facets, where each facet contains four hexagrams, four facets would fit together into a hyperfacet, and the four hyperfacets would fit together to form the complete lattice. In fact, any division of the lattice into a set of equal-sized chunks can serve as an unfolding — providing as we have said, that each chunk is a coherent group of hexagrams and the chunks themselves reflect the overall structure of the lattice.

The mathematical description of an unfolding is given in terms of a hexagram defining the change quanta governing the whole unfolding. The sub function is used to define the individual substructures within the unfolding as follows:

The unf Function

$$\text{unf}(q) = \{\text{sub}(g, q) : g \in \text{sub}(0, \sim q)\}$$

This states that an unfolding relative to q is a set of substructures, each with the change quanta q applied to some hexagram g . The hexagrams g , which provide the basis of each substructure in the unfolding, are drawn from the Boolean inverse of q applied to Receptive. An example will help to make this clear. Suppose we are interested in the unfolding defined by taking the hexagram Peace as the change quanta q . Then, the set of hexagrams defined by $\text{sub}(0, \sim q)$ — that is, the substructure generated by applying Standstill to Receptive — would be:



These eight hexagrams then serve as the root of each of each of the cubes within the unfolding. The complete cubic unfolding for Peace is shown at the end of the paper; each of the hexagrams shown here appear as the base of one of the cubes in that unfolding. That is, in

$\text{unf}(q)$ where q is Peace, the g in $\text{sub}(g, q)$ is alternately Receptive (shown in Figure 4 above), Enthusiasm, Gathering Together, Holding Together, Contemplation, Standstill, Progress, and Splitting Apart. Each hexagram, combined with Peace, generates each of the cubic substructures in the unfolding.

We saw when considering sub , that more energy in q gives more complex substructures. Because unf is based on sub , the same situation arises here: the more energy there is in q , the more complex the substructure components of the unfolding are. Following on from this, the more complex the substructure components in the unfolding are, the fewer of them there will be overall. So, a low energy q gives an unfolding with many, simple substructures, whilst a high energy q gives an unfolding with fewer, more complex substructures. This is described fully in the following table:

Energy in q	Size of Substructure		Seq Len	Description
	Dim	Hex		
0	0	1	64	Linear Unfolding
1	1	2	32	Pair-wise Unfolding
2	2	4	16	Planar Unfolding
3	3	8	8	Cubic Unfolding
4	4	16	4	Hypercube Unfolding
5	5	32	2	Lattice Bifurcation
6	6	64	1	Complete Lattice

Table 6: The Different Types of Unfolding

The first column shows the energy in q , the number of yang lines that the hexagram contains. The next two columns show the size of the resulting substructures, firstly in terms of the number of dimensions, which is always the same as the energy in q , and secondly in terms of the number of hexagrams in each substructure. The fourth column shows the number of such substructures that there are in the full unfolding and the final column gives a name for the type of unfolding.

The following interpretation of this is suggested: if there is no energy in q , that is, if awareness is not focussed on any dimension of change, then one will only see each single situation as it arises, the current moment. In this way, one can engage fully with the situation, but there will be no context to help understanding. The result is that experience unfolds as a linear sequence of situations. As one expands one's awareness to include more dimensions of experience, so the amount of context available for consideration in any given situation increases. Then, experience seems less like a linear sequence of situations and more like a connected web of situations, unfolding as one progresses. Ultimately, if one were able to maintain simultaneous awareness of all the dimensions of experience, then one would be able to see the complete context of a situation. We shall now consider some examples.

Linear Unfolding

Even in the context of multi-dimensional lattice theory, a linear sequence of symbols with a coherent ordering principle presents a useful way to structure the symbols. The most natural sequence for the Boolean lattice is known as a Hamiltonian Cycle:²³ starting from Receptive, this visits each hexagram in turn once, without any repetition and returning to Receptive at the end. This forms a great spiralling loop through the lattice. If you start with Receptive in the

cubic unfolding diagram shown at the end of the paper, and follow the arrows through the structure, then the sequence is a Hamiltonian Cycle through the lattice. Let us consider the simpler trigram lattice in detail, which will form the basis for the path through the hexagrams. The following sequence of gua shows a type of Hamiltonian Cycle known as a reflected path:



Trace this through the trigram lattice to see how each step of the sequence is a single step in the lattice. Notice the spiralling path taken by the sequence. From an energetic point of view, such sequences are of particular interest because, at each step, only one line changes. This means that each step is done with the minimal energy change possible and connectedness is maintained.

As we have seen in Table 6 above, if Receptive is taken as the q parameter to unf, then the substructures collapse down to individual hexagrams. In that case, the Hamiltonian Cycle discussed here is used as the basis for sequencing them. This cycle is also used as the basis for sequencing the more complex substructures in the unfoldings explored below.

Planar Unfoldings

Planar Unfoldings deconstruct the lattice by taking one facet at a time and linking them together. Each facet is two-dimensional face formed as the result of changing energies in two lines; depending on which two lines are chosen, radically different unfoldings can result. Two natural unfoldings arise by considering first Preponderance of the Small and secondly Corners of the Mouth as values for q . Preponderance of the Small makes the two central lines of the hexagram the focus for change, whilst Corners of the Mouth makes the top and bottom lines the focus for change. The first four facets for each unfolding are shown below, and the complete unfolding for Preponderance of the Small is shown as Figure 8 at the end of the paper.

If we inspect the first facets of each of these unfoldings, an interesting fact emerges: the first structure in every unfolding sets the scene for the whole unfolding. First, note that the bottom hexagram in the facet is Receptive, this is the zero-energy starting point of all unfoldings, and it is always the bottom element of the first structure in any unfolding. Secondly, note that the top hexagram in the first facet is the q hexagram which defines the energy of the unfolding as a whole. This is also always the case: the top hexagram in the first structure of any unfolding is the q value of that unfolding.

Turning now to these specific examples. If we consider the meanings of the different lines discussed earlier, then the first unfolding shown is the result of focussing awareness in heart and mind, in consciousness itself. This represents a natural unfolding of situations and contexts for someone taking an introspective approach to life. Modesty and Enthusiasm are the necessary components for progress. In contrast, the second unfolding is focussed on matter and pattern, the two extreme poles of reality; these are the concerns of philosophy and science, trying to identify the patterns and structures of the world. Return and Splitting Apart are the two components and these are the first and last steps of the underlying Hamiltonian Cycle; thus, this facet unites the beginning and ending.

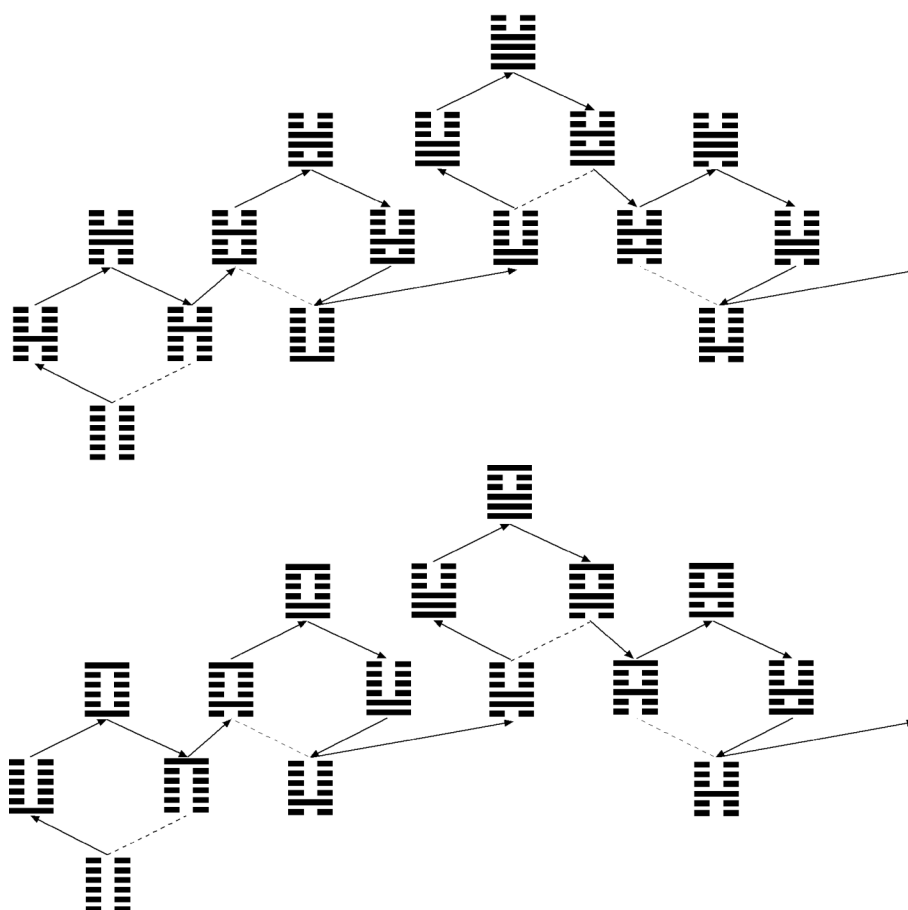


Figure 5: Two Partial Examples of Planar Unfoldings

We have noted that Receptive is, by definition, the first hexagram in all unfoldings. However, most of the rest of the patterns are quite distinct – notice that the first facet identifies the lines which are in focus in each case, whilst subsequent facets show those changes in developing contexts. In the case of these two unfoldings, Peace appears in the same place in both, but this is not generally true. However, it is the case that Creative always appears in the same place in all unfoldings; in the case of planar unfoldings it is always at the apex of the 11th facet, in the case of cubic unfoldings it is always at the apex of the 5th cube. Thus Receptive and Creative retain their role as the primary axis of energy even when the lattice is unfolded.

Cubic Unfoldings

A cube is a natural way of relating to space and, once its relationship to the lattice of trigrams is understood, it can be used as an effective basis for exploring the structure of the complete hexagram lattice. A cubic unfolding will break the lattice down into eight connected cubes: each individual cube is a three-dimensional structure, and each is connected to the next and previous cubes in the sequence by a collection of higher dimensional links, with the sequence of cubes forming a continuous loop, coiling back to the starting point. Planar unfoldings appear, wrapped inside the cubic unfolding; pair-wise unfoldings appear wrapped inside the planar unfoldings.

There are 20 possible cubic unfoldings of the whole lattice, corresponding to the 20 hexagrams with three yang lines. Four of these present themselves for immediate

consideration, focussing on the four different constituent trigrams that can be identified within a hexagram:

- (i) Considering the bottom trigram: the focus of awareness is on the possible configurations of explicate matter. This corresponds to unf applied to Peace.
- (ii) Considering the lower nuclear trigram: the focus of awareness is on the energies of manifest being. This corresponds to unf applied to Duration.
- (iii) Considering the upper nuclear trigram: the focus of awareness is on the energies of transcendent being. This corresponds to unf applied to Influence.
- (iv) Considering the upper trigram: the focus of awareness is on the implicate patterns. This corresponds to unf applied to Standstill.

As for the planar unfoldings, each of these cubic unfoldings gives rise to a distinct perspective on the relationships between the hexagrams, emphasizing some connections at the expense of others; this is the price of reducing the structures to comprehend them. Note that the cubic unfoldings listed here are the only four cubic unfoldings where the focus of awareness is on a coherent domain (i.e. the lines in focus are adjacent).

The complete cubic unfolding shown in Figure 9 at the end of this paper corresponds to (i) above. If we think of this in terms of Bohm's explicate and implicate orders, then the cube presented in focus at any particular moment would represent the current state of manifest reality, the explicate order, with all of the changes manifesting in the lower three lines set against the given implicate energy, whilst the other seven cubes would represent the rest of the implicate order, aspects of reality waiting to become manifest through the process of further unfolding implicate patterns.

Other Unfoldings

We have looked at a number of different unfoldings of the hexagram lattice, all based on the same mathematical definitions of sub and unf. Initially, we considered the Receptive linear unfolding to give a simple sequence of hexagrams that formed a cycle through the lattice, with each step representing a minimal energy change. We then saw two-dimensional subspaces giving a flat unfolding of the lattice, like peeling it. Finally, we also considered cubic subspaces giving a solid unfolding of the lattice.

It is also possible to unfold the lattice into four- and five-dimensional subspaces. However, the utility of such unfoldings is initially less than those shown here. This is because all of the techniques shown above reduce the hexagram to a series of three, or fewer, dimensions, and makes the total structure more comprehensible at the expense of simplification. Higher dimensional unfoldings do not initially provide sufficient simplification to bring the lattice structures within our normal cognitive space. However, once one is familiar with the technique, higher dimensional unfoldings offer a way of extending the visualization. For example, it is possible to construct a four-dimensional unfolding that contains pairs of cubes from a three-dimensional unfolding. Also, five-dimensional unfoldings divide the total lattice into halves, allowing us to focus on all the hexagrams with say, a yin line in the first place and then the complementary set of hexagrams with a yang line in that place.

What are we to make of this multiplicity of perspectives? Birdwhistell²⁴ says that "Shao [Yung's] presentation is a demonstration that no single perspective can enable one to observe

all facets of the universe adequately.” Now we have a mathematical way of representing multiple perspectives directly in the organization of the gua.

Interpreting Structures

Just as the algebraic operators themselves are interpreted, so the unfolded lattice structures generated by the algebra, and the substructures within them, should all be given meaning within any cosmology of the Yijing. Although the algebraic formulations carry intrinsic meaning in virtue of their mathematical properties, it is useful to explore the addition of a narrative to the diagrams. I have suggested some possible interpretations for the various unfoldings shown above, and it is interesting to follow up with an example in more detail. Consider the two-dimensional unfolding for Preponderance of the Small in Figure 8. Let us look at the fifth facet in that unfolding:

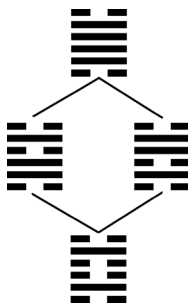


Figure 6: Facet for Opposing Tensions

This is an introspective unfolding and, for each facet, the middle two lines of the hexagram change whilst the top and bottom pair of lines stay stable, exploring the effects of adopting different attitudes within particular situations. I call this particular facet *Deciding Attitude to Opposing Tensions*. Notice that in all four hexagrams the lower bigram, in terms of the lattice cycle, is Ending and that the upper bigram is Beginning; this creates an opposition of forces. The differing attitudes that the individual may take towards these forces are then represented by allowing the middle bigram to take on any of the four possible energy states.

The question then arises “how does the individual at the centre handle the forces exerted on them?” The elements of this square are: Preponderance of the Great at the top in the full place; The Well in the middle left, the beginning place; Oppression in the middle right, the ending place; and Abysmal at the bottom, in the empty place. Let’s start at the top and work down.

Preponderance of the Great is full in the centre, and consciousness has its maximum energy. This represents trying to satisfy both external forces at once, and it requires a constant output of energy from the individual in both directions. The traditional image for this hexagram is of a ridgepole supporting a roof. The beam is weak at the ends, and the load imposed is in danger of collapsing the supports. Trying to constantly satisfy two conflicting demands is certainly a situation that is at the limits of what can be sustained. If you are weak, you will be pulled in two. If you are strong, you will be exhausted. It is necessary to establish a direction.

Both of the next two states have a lower energy level, but involve taking a particular direction. If you align your attitude with the pull of the energy in the upper bigram, then the central bigram takes the same character; the resulting hexagram is The Well. This is the

wellspring, the unchanging source of communal goodness, which must be maintained. Regardless of what happens in a community, if the well is looked after, then the community will flourish. So, aligning oneself with the budding implicate energy, and adding to that energy by adopting the same attitude oneself, will create a sustainable situation of great potential. The energetic alternative to aligning with the upper bigram, is to align with the lower. If you intentionally still yourself, reinforcing circumstance, then the result is Oppression. This results in being enclosed and trapped, in being exhausted by the circumstances. To succeed from this position is difficult, and a change of context would be recommended.

If you avoid the choice of direction, and empty yourself completely of involvement with the outcome, then you find yourself in Abysmal. Your only function here is to anchor the two opposing forces together and therefore, you only serve to perpetuate their relative tension. Further, because their forces are transmitted through you, you too are held in a state of tension. Sometimes, maintaining emptiness relative to the surrounding forces can absolve fault in a situation, but not here. Here it is dangerous for all concerned.

Clearly, given the large number of possible unfoldings, and the large number of structures within them, providing a narrative description of all aspects of all possible perspectives is a daunting undertaking. The most important aspect of this work lies in the general principles that generate the unfoldings, and the interpretation of those principles in cosmological terms. However, it is interesting and important to check the results of the mathematical analyses by considering the meanings of the hexagrams in the context provided by an unfolding.

Conclusions

It is clear that the Yijing has been understood as being about the entokening of abstractions since early in its history. Successive generations of sages have been reflecting on its nature, and the opening chapters of the *Da Zhuan* speak to this theme directly:²⁵ Chapter One says “Events are grouped to type. Things are divided into classes.” This is abstraction at work. Further, the symbolic means through which these groupings are described are the *gua*, and Chapter Two goes on to say “the sages devised the *gua* so that images could be seen therein.” Wang Bi²⁶ expands on this in the following way: “since the words are the means to express the images, once one gets the images, he forgets the words, and, since the images are the means to allow us to concentrate on the ideas, once one gets the ideas, he forgets the images.” Thus, we see that the Yijing presents us with a related series of abstractions at different levels, each level serving as the point of entry to the next.

This paper has sought to explore the underlying cosmology of the Yijing from within the contemporary language of quantum physics, logic and algebra. I have shown that the *gua* can be seen as a system of binary categories, and suggested that they classify abstract situations using a spectrum of energies ranging from the concrete to the abstract. I have argued elsewhere that exploring correspondences between different systems of thought is best done at the deepest conceptual levels of both systems, rather than through superficial correspondences in the surface symbolism.²⁷ Following this, I have suggested that an important concept from quantum physics, that of Bohm’s implicate and explicate orders, has a direct connection with the classical cosmology of the Book of Change. It is interesting to note that there is another deep connection between the Yijing and quantum physics: in both systems randomness plays an important part. In quantum physics, the underlying processes of particle behaviour are deemed to be random. In the Yijing, the divination process makes use of random events.²⁸

In applying Boolean algebra to the gua, a complex six-dimensional structure is revealed which can be seen as representing the totality of reality. In order to make this structure more comprehensible, ways of unfolding it into sets of simpler structures were explored. These techniques suggest that the act of focusing our awareness on different aspects of the situations we encounter results in different perspectives, different views of the world. If Wang Bi's analogy is followed through, if the textual components associated with the symbols are descriptions of images, and those images represent ideas, then the structural relationships between the gua should also find some meaning in this context. The role of the Boolean lattice is to define the space in which ideas exist. The structural relationships between the gua then express the relationships between the ideas. These structural relationships create subspaces within the lattice, and are themselves open to a narrative description.

It is important that the new techniques made available through the algebraic approach are in harmony with the cosmological underpinnings of the Yijing. To this end I have explored the philosophical implications of applying Boolean analysis to the gua by comparing some of the formal properties of the resulting structures with ideas from the texts of the *Shou Gua* and *Da Zhuan*. Further, the presence of similar diagrams in collections such as the *Zhouyi Tuishi Dadian* shows that some thought has already moved in this direction.

In conclusion, this paper suggests that the Yijing is constantly developing and expanding. As each new culture encounters and embraces it, it absorbs new aspects whilst retaining its core principles. The current phase, incorporating various formal-structural analyses, arises as a natural step – part of the continuous existence of the book through into the age of computational representations.

Lattice Diagrams

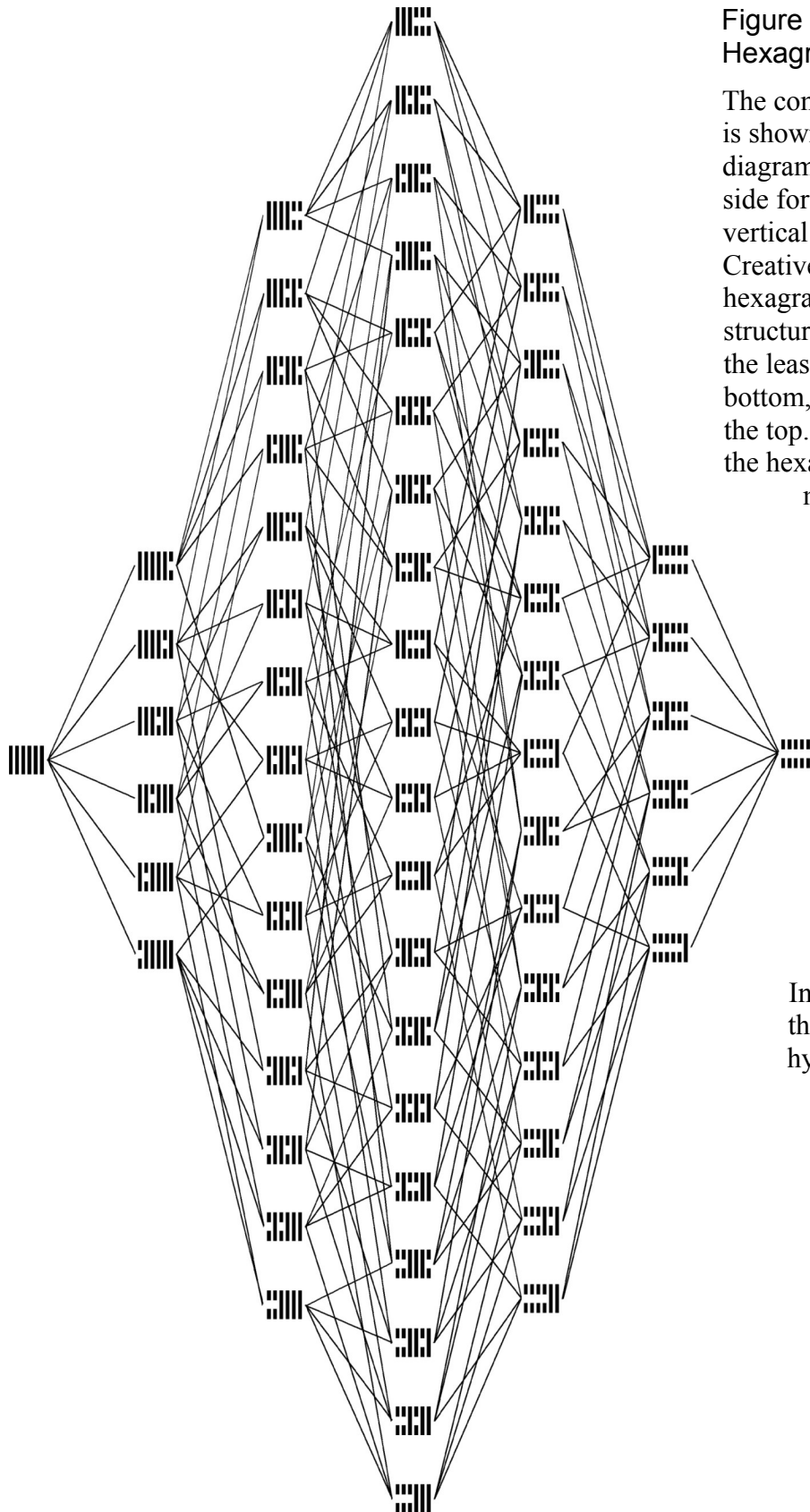


Figure 7: The Complete Hexagram Lattice

The complete hexagram lattice is shown here. Note that this diagram has been turned on its side for convenience, and the vertical axis actually runs from Creative to Receptive. All hexagrams are arranged in the structure by energy level, from the least energetic at the bottom, to the most energetic at the top. The lines connecting the hexagrams show the minimal energy changes from one situation to another.

In mathematical terms, this is a six-dimensional hypercube.

Figure 8: Example Planar Unfolding

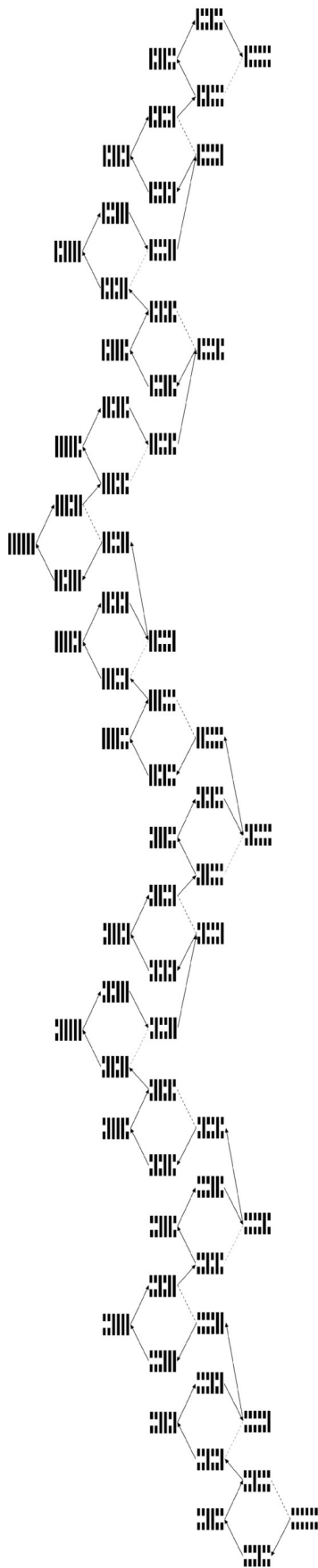
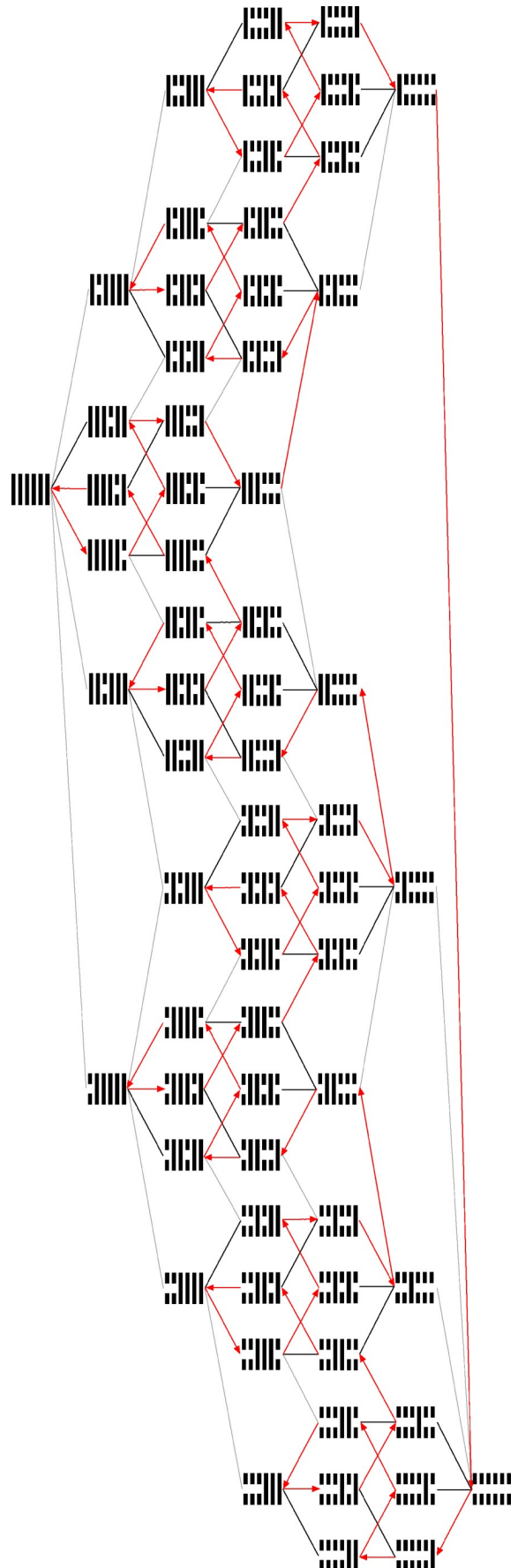


Figure 9: Example Cubic Unfolding



¹ Some of the formal background for the work presented here was originally presented in my paper “Boolean Algebra and the Yi Jing” which appeared in *The Oracle: the Journal of Yijing Studies*, Volume 2, Number 7, Summer 1998, pp19–34. This paper is available for download from <http://www.yijing.co.uk>

² See Anne D. Birdwhistell’s book *Transition to Neo-Confucianism: Shao Yung on Knowledge and Symbols of Reality*, published by Stanford University Press, California, 1989.

³ *Zhouyi Tuishi Dadian* “Encyclopaedia of Zhouyi Diagrams” published in two volumes in Beijing in 1994. Unfortunately, I do not have direct access to this material and I am indebted to S. J. Marshall for making scans of some of these diagrams available at <http://www.biroco.com/yijing/scan.htm>.

⁴ Thomas Cleary’s 1989 book *I Ching Mandalas: A Program of Study for the Book of Changes*, published by Shambala Publications Inc. contains a collection of diagrams similar to those in the *Zhouyi Tuishi Dadian*.

⁵ Sung’s work was originally published in 1934 as *Symbols of the Yi King or The Symbols of the Chinese Logic of Changes*. The version that I have seen is from the Paragon Print Reprint Corp. 1969.

⁶ Daniel S. Goldenberg, “The Algebra of the I Ching and its Philosophical Implications”, in *Journal of Chinese Philosophy* 2, 1975 pp149–179. D. Reidel Publishing Company.

⁷ The algebraic properties of nuclear hexagrams are explored in the paper “Tools for Change”, co-authored with Stephen Karcher. This appeared in *The Oracle*, Volume 2, Number 12, January 2002. This joint paper is available as a download from <http://www.yijing.co.uk>. A paper exploring the algebraic properties of the different forms of opposition operator is in preparation.

⁸ William de Fancourt’s book *Warp and Weft: In Search of the I-Ching*, published by Capall Bann Publishing, Chieveley, 1997, contains a good account of the Yi’s early contact with the west.

⁹ Jon Barwise and John Perry *Situations and Attitudes*. Published by Bradford Books, the MIT Press, Cambridge, Massachusetts, 1983.

¹⁰ Kirk McElhearn’s excellent paper “The Key to the Yi Jing” (*The Oracle: the Journal of Yijing Studies*, Volume 1, Number 6, Spring 1998) explores the idea the hexagrams are situations from an informal perspective.

¹¹ David Bohm, *Wholeness and the Implicate Order*, Routledge Classics, London, 2002.

¹² Quotes from the *Da Zhuan* and *Shou Gua* come from the translation of the Yi Jing by Wu Jing-Nuan, published in 1991 by The Taoist Center, Washington D.C. This quote is from page 282.

¹³ Marysol Gonzales Sterling, *I-Ching and Transpersonal Psychology*, published by Samuel Weiser, Inc., York Beach, Maine. 1995.

¹⁴ Thomas Cleary, 1989 *ibid.*, pages 21–26.

¹⁵ For those readers wishing to pursue the mathematics of lattice work further, a good place to start is *Introduction to Lattices and Order* by B. A. Davey and H. A. Priestly, published by Cambridge University Press, 1987.

¹⁶ For the sake of familiarity I shall use the English translations of the hexagram names from Richard Wilhelm’s book *I Ching or book of changes*, published by Routledge and Kegan Paul, London, 1983.

¹⁷ In general, it is possible to represent all of the changes of the Yijing within the different components of the complete lattice. This gives a technique for categorising the Forest of Change, which I plan to explore in a latter paper.

¹⁸ The King Wen sequence is actually organized into pairs of hexagrams, but we shall ignore this aspect in this presentation.

¹⁹ A good reference for basic logical calculi is E. J. Lemmon's book *Beginning Logic*, published by Thomas Nelson and Sons, London, 1965. In this paper I use Lemmon's notation for the operators: the "and" operator is represented by the "&" symbol, and the "or" operator is represented by the symbol "v" (from the latin word "vel").

²⁰ My paper "Flowers and Steps in the Boolean Lattice of Hexagrams" describes how the changing lines in a hexagram can be interpreted as a journey through a subspace of the complete Boolean lattice. This appeared in *The Journal of Chinese Philosophy*, Volume 31, Number 4, December 2004, pp489–504. This is also available for download from my web site, <http://www.yijing.co.uk>.

²¹ *The Official String Theory Web Site* is an excellent source of information explaining the basics of string theory. It can be found at <http://superstringtheory.com>

²² Some authors have explored the suggestion of extending the symbols of the Yijing to higher dimensions. In particular, Salyer and Leal suggest adding a third trigram to the gua, making the system nine dimensional. See Salyer and Leal, *Omei I Ching: Mechanics of the I Ching*, published by The Watercress Press, San Antonio, 1999. I am currently working on a paper which explores this possibility, suggesting that after 6 lines, the next *useful* size for hexagrams is 12 lines.

²³ The first person to use this idea to order the gua was Peter Loly in his article "A Logical Way of Ordering the Hexagrams" which appeared in Volume 2, Number 12 of *The Oracle*, January 2002, pages 2-13. He refers to the path as Gray Code, which is how it is known in an alternative branch of mathematics.

²⁴ Birdwhistell, *ibid.*, page 96.

²⁵ Wu Jing-Nuan, *ibid.*, page 263.

²⁶ Quotes from Wang Bi come from Richard John Lynn's 1994 translation *The Classic of Changes: A New Translation of the I Ching as Interpreted by Wang Bi*. Published by Columbia University Press, New York.

²⁷ My paper "Great Polar Fist and the Book of Change" which originally appeared in the *Journal of the Tai Chi Union for Great Britain*, Number 18, Spring 2004, pp14–17, explores the deep conceptual connections between the art of Taijiquan and the Yijing. This is also available for download from my web site, <http://www.yijing.co.uk>.

²⁸ I explore the nature of randomness in my paper "On the Nature of Divination", originally published in the *Journal of the Organisation for Counselling and Psychotherapy in Scotland*, in the Spring/Summer 2004 edition. This is also available for download from my web site, <http://www.yijing.co.uk>.