

The Yijing as a Symbolic Language for Abstraction*

Dr Andreas Schöter

Introduction

The symbols of the Book of Change present an astonishing enigma. Their origins lie deep in China's past, born from the divinatory practices of the Zhou dynasty – but the patterns of open and closed lines resonate down through the millennia to the present day. This paper looks at the symbolic language of the Book of Change, applying techniques from Boolean algebra to generate and analyze structures which represent the relationships between the symbols.¹ This process is part of an intellectual lineage that began well before the Yijing had contact with the West. Shao Yung's work on the binary sequence of the symbols² and the various diagrams in the *Zhouyi Tuishi Dadian* make this clear.^{3,4} More recently this thread was continued in Z. D. Sung's geometric characterizations from the 1930s.⁵ The first modern mathematical work on the Yijing in the West that I am aware of is by Goldenberg in 1975 who bases his approach on modulo arithmetic.⁶ In contrast, the algebra used here comes directly from computer science and integrates with the nature of the symbols of the book.

This paper presents a mathematical way of representing multiple perspectives on reality. The whole of reality is too complex and detailed to be grasped as one. Instead, each of us sees a partial view from our own perspective. By starting with the idea that the symbols of the Yijing encompass the totality of reality, we shall see that by focusing awareness on different dimensions of experience, by concentrating on different abstractions, different perspectives naturally unfold. This idea is given a precise formal presentation, by showing how the Yijing can be seen as a symbolic language, and then exploring the algebraic properties of that language.

Abstraction and Symbolic Language

Before diving into the details we should first explore what is meant, in this context, by the terms *abstraction* and *symbolic language* and how they are applied to the Yijing.

Abstraction

The ability to abstract common features from diverse situations is a key cognitive function, vital in the formation of concepts and our day-to-day functioning. When we look at a ball, a hoop, a wheel, and the full moon we might recognize something that they have in common: namely, roundness. But what is "roundness"? We cannot point to any particular *thing* in the world and say "that is roundness"; we can only point to particular objects and say "that thing has the property of roundness". Roundness is a concept that we abstract as a property from a wide range of concrete objects which exhibit it.

Seeing what is common in things that appear different enables us to generalize about situations and apply techniques learnt in one set of circumstances to problems encountered in another. We can understand the symbols of the Yijing as representing abstract concepts that

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can be applied to novel situations as they are encountered. The symbols give us a set of categories that we can use to organize and interpret our experience.

Symbolic Language

What do we mean by a *symbolic language*? In the context of this discussion, we shall take it to be composed of some fixed set of symbols (the words of the language), and some rules for transforming between symbols and for combining groups of symbols into structures (the grammar of the language). The symbols themselves are taken to have meaning, where this meaning comes about by in part by virtue of reference to objects in some domain being symbolized. The rules themselves should also have meaning, representing processes and structures in that domain.

Enumerating all the possible combinations of open and closed lines over six places, the 64 hexagrams can be seen as the first systematic, symbolic language for categorization. The hexagrams themselves are the symbols of the language. The traditional method of representing changing lines, which generates relationships between pairs of symbols, is one example of a rule of symbol transformation.

The traditional domain of application for the symbols of the Yijing could be described as the study of the interaction between human consciousness and its environment. That is, the hexagrams provide us with a binary notation for describing the interactions of the various forces that are at work in the universe, and our place relative to those forces. The narratives attached to the symbols record the traditional interpretations of those interactions, but the trigrams and hexagrams themselves are also taken to provide a direct, iconic image of the forces at work.

A Graphical Introduction to Lattices

We have seen that an algebraic system consists of a set of symbols (the *gua*) and a collection of operators that act on those symbols to combine and transform them. Those operators generate a set of structures that organize and classify the *gua*; these structures are lattices.⁷

One-Dimensional Lattices

There is a very natural connection between the complexity of the structures generated by a Boolean algebra and the geometric idea of *dimensionality*. In what follows we shall start with single line structures and build up to hexagrams. So, for single lines, the lattice shown in Figure 1(a) below is generated:

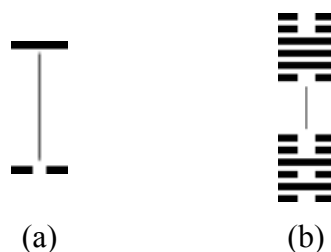


Figure 1: One-Dimensional Lattices

This shows the two possible states that a single line might be in and the energetic relationship between those two states. Thus, a line might be either yin or yang, and a yang line has more energy than a yin line. In terms of information theory, we would say that this represents the

two possible situations that a single binary category can distinguish, it is the basic polarization. In geometric terms, this is a one-dimensional structure, or an *edge* in more complex structures. For example, the one-dimensional lattice shown as Figure 1(b) connects the hexagrams Duration and Deliverance⁸ with the lattice edge being formed by the energetic difference in the third line. Note that there are 192 edges in the complete hexagram lattice, which encode the 384 single line changes of the Yijing.⁹

Two-Dimensional Lattices

When we consider pairs of lines, the lattice shown in Figure 2(a) below is generated. This shows the energetic relationships between the four possible states that can arise from two lines. At the bottom, analogous to Receptive, is the pure yin bigram with the lowest energy level; and at the top is pure yang, Creative with the maximum energy level. On the left, analogous to Thunder is a yang line bursting forth beneath yin, beginning the cycle around the lattice. On the right, analogous to Mountain, is a yang line resting above yin, ending the cycle around the lattice. When moving between any two adjacent gua in the structure, only a single line of energy changes. Thus, to borrow a term from physics, each edge of the lattice describes a quantum state change between two situations. Geometrically, this lattice for bigrams is two-dimensional and we will see that, projected up to hexagrams, it generates facets of the whole lattice.

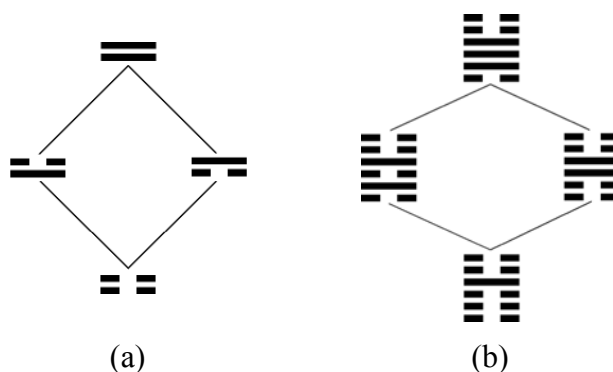


Figure 2: Two-Dimensional Lattices

Consider the two-dimensional structure shown as Figure 2(b), this is created as the result of the energetic differences in the second and third lines of the hexagrams: notice that the edge between Duration and Deliverance shown in Figure 1(b) appears here as one of the edges of the facet. There are 240 unique facets in the complete hexagram lattice, encoding the 960 changes involving two lines. We will see next how this facet itself appears as part of a cube.

Three-Dimensional Lattices

Finally, we consider the cubical lattice for trigrams shown in Figure 3(a) below. As before, an interpretation of the structure in terms of energy state is natural. Receptive is at the bottom, with no yang energy. In contrast, Creative is at the top, with maximum yang energy. The other six trigrams are arranged in layers between, with first one line of yang energy, and then two. The lattice represents the various energy states of situations, defining the field of quantum potential for trigrams, and the connections within the lattice show the minimal transitions from one state to another. Geometrically, the trigram lattice is a three-dimensional structure, which each trigram connected to others through the changes of its lines.

Now consider a corresponding three-dimensional figure for hexagrams shown as Figure 3(b). We see a cube of hexagrams, formed as the result of the energetic permutations of the lower

trigram. The hexagram Power of the Great is at the top and Enthusiasm is at the bottom. Notice that the two-dimensional lattice shown as Figure 2(b), between Enduring and Enthusiasm, appears as a facet of this cube. There are 160 unique cubes in the hexagram lattice encoding the 1280 changes involving 3 lines.

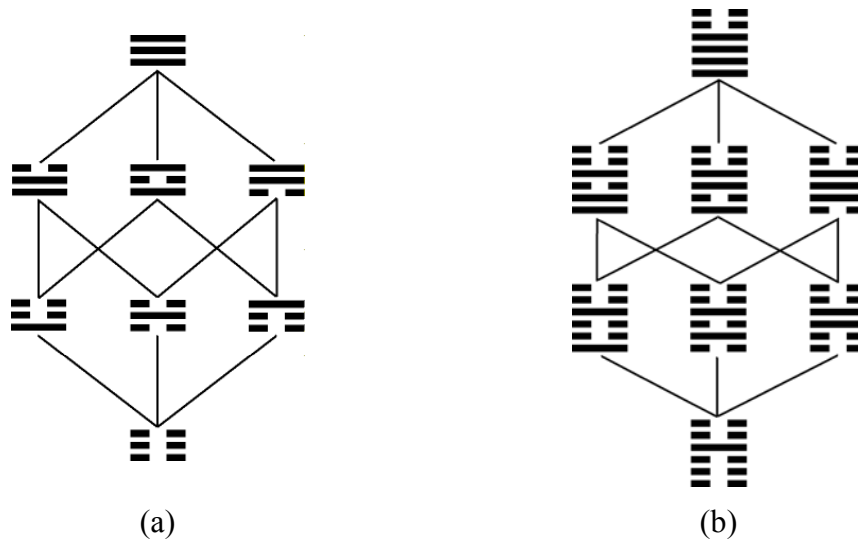


Figure 3: Three-Dimensional Lattices

So, we can see that the structures created at one level of dimensionality are projected upwards into the higher dimensional structures. The lines of the one-dimensional lattice form the edges of two- and three-dimensional lattices. The squares of the two-dimensional lattice form the facets of the three-dimensional lattice. This pattern continues up through the higher dimensional structures until the complete six-dimensional lattice of the hexagrams is reached. This is a complex structure which is hard for our three-dimensional visual minds to grasp in its totality (see Figure 5 at the end of the paper). We shall now see how it can be made more comprehensible by unfolding it to different degrees.

How Everything Unfolds

We will now explore how to unfold the six-dimensional hexagram lattice into a variety of simplified structures.

The Multidimensional Nature of Reality

We have seen that, following the geometric story, the Boolean algebra generates a six-dimensional lattice structure when applied to hexagrams; this is the fractal repetition of the three-dimensional trigram cube up to the higher dimensional space of hexagrams.

The idea that reality consists of more than the three-dimensional space that our senses reveal to us can be understood in many different ways. It is a conjecture explored in contemporary physics through the topic of string theory. This theory postulates that there are additional dimensions of reality, folded up within the normal three that we are used to experiencing. Most versions of string theory postulate 10 dimensions in total; however, the exact number of dimensions is of less interest than the idea itself.

In the case of the Yijing, the dimensions that we are representing are not only literal physical dimensions, but also dimensions of experience, emotional and psychological dimensions, dimensions of thought and abstraction. We shall see below, by exploring different ways of

unfolding the hexagram lattice, that different ways of focussing our attention result in very different perspectives on reality.

Substructures

It is possible to define a computational function which can be applied to hexagrams to generate the substructures shown above.¹⁰ Let us call this function $\text{sub}(p, q)$, where p is the primary hexagram and q is the change quanta applied to the primary hexagram to generate the structure.¹¹ Some examples will make its application clear, and the following table shows the functions which would generate the lattice substructures shown above.

Substructure	Dimensions	Function
Figure 1(b)	1	$\text{sub}(\text{Hexagram 1}, \text{Hexagram 2})$
Figure 2(b)	2	$\text{sub}(\text{Hexagram 1}, \text{Hexagram 2})$
Figure 3(b)	3	$\text{sub}(\text{Hexagram 1}, \text{Hexagram 2})$

Firstly consider the edge lattice shown in Figure 1(b), this would be generated by the application of the sub function shown in the third column. Here, Deliverance is the primary hexagram and Modesty is the change quanta. In this case q has a single yang line in the third place, so this is where the change is focussed, resulting in a simple edge. In the case of Figure 2(b) q has two yang lines in the second and third place, resulting in a square based on Enthusiasm. Similarly, in the case of Figure 3(b) the primary hexagram is Enthusiasm, but the change quanta is Peace. Because Peace has three yang lines in the lower trigram, this is where the change is focussed, resulting in a cubic substructure.

Note that cubes are always formed by a sub function with a q parameter chosen from the 20 hexagrams with three yang lines. Similarly, two-dimensional facets are always formed from a sub function with a q parameter chosen from the 15 hexagrams with two yang lines. In general, if the q parameter to the sub function has n yang lines, then the result will be an n -dimensional subspace. So, the more energy there is in q , the more complex the resulting substructure it creates. The sub function gives us an easy way of creating the full range of subspaces. If we chose Receptive for q , with no changing lines, then we will get a zero-dimensional subspace, that is, the single hexagram corresponding to p ; if we chose a hexagram with one changing line, for example Return, then we will get a one-dimensional subspace and so on. If we chose Creative, then we get a six-dimensional subspace which is, of course, the complete lattice itself.

Unfolding the Complete Lattice

The purpose of unfolding the complete lattice is to break it down in to manageable substructures, where each substructure is easier to understand than the totality. Also, because conscious awareness itself is typically partial and changing, these unfoldings are a better conceptual representation of our actual experience. By using the appropriate mathematical definitions, we can define a set of substructures which, taken together, form a natural sequence that covers all of the lattice, and where each substructure within the sequence provides a coherent grouping of hexagrams, reflecting the overall structure of the lattice. That is, each unfolding provides a holographic fragmentation of the whole lattice, mirroring its structure at different degrees of dimensionality. A diagrammatic example of a complete unfolding is shown as Figure 6 at the end of the paper.

For example, we could unfold the lattice into 16 facets, where each facet contains four hexagrams, four facets would fit together into a hyperfacet, and the four hyperfacets would fit together to form the complete lattice. Alternatively, we could consider unfolding the complete lattice into eight cubes, where each cube contains eight hexagrams, and all the cubes taken together cover the complete lattice, fitting together into a larger hypercube. In fact, any division of the lattice into a set of equal-sized chunks can serve as an unfolding – providing, as we have said, that each chunk is a coherent group of hexagrams and the chunks themselves reflect the overall structure of the lattice.

The mathematical description of an unfolding is given in terms of a hexagram defining the change quanta governing the whole unfolding. The sub function is used to define the individual substructures within the unfolding.¹² This states that an unfolding relative to q is a set of substructures, each with the change quanta q applied to some primary hexagram p . The hexagrams p , which provide the basis of each substructure in the unfolding, are drawn from the Boolean inverse of q applied to Receptive. An example will help to make this clear. Suppose we are interested in the unfolding defined by taking the hexagram Preponderance of the Small as the value of the change quanta q . Then, the set of hexagrams defined by $\text{sub}(0, \sim q)$ – that is, the elements of the substructure generated by applying Inner Truth to Receptive – would be:



These 16 hexagrams then serve as the root of each of the facets within the unfolding. Each base, combined with Preponderance of the Small, generates each of the cubic substructures in the unfolding. See the upper sequence in Figure 4 for the first four facets built on this sequence.

Planar Unfoldings

Planar Unfoldings deconstruct the lattice by taking one facet at a time and linking them together. Each facet is a two-dimensional face formed as the result of changing energies in two lines; depending on which two lines are chosen, radically different unfoldings can result. Two natural unfoldings arise by considering first Preponderance of the Small and secondly Corners of the Mouth as values for q . Preponderance of the Small makes the two central lines of the hexagram the focus for change, whilst Corners of the Mouth makes the top and bottom lines the focus for change. The first four facets for each unfolding are shown below, and the complete unfolding for Preponderance of the Small is shown as Figure 6 at the end of the paper.

If we inspect the first facets of each of these unfoldings, an interesting fact emerges: the first structure in every unfolding sets the scene for the whole unfolding. First, note that the bottom hexagram in the facet is Receptive, this is the zero-energy starting point of all unfoldings, and it is always the bottom element of the first structure in any unfolding. Secondly, note that the top hexagram in the first facet is the q hexagram which defines the energy of the unfolding as a whole. This is also always the case: the top hexagram in the first structure of any unfolding is the q value of that unfolding.

Turning now to these specific examples. The first unfolding shown is the result of focussing awareness in heart and mind, in consciousness itself. This represents a natural unfolding of situations and contexts for someone taking an introspective approach to life. Modesty and Enthusiasm are the necessary components for progress. In contrast, the second unfolding is

focussed on matter and pattern, the two extreme poles of reality; these are the concerns of philosophy and science, trying to identify the patterns and structures of the world. Return and Splitting Apart are the two components and these are the first and last steps of the traditional seasonal cycle; thus, this facet unites the beginning and ending.

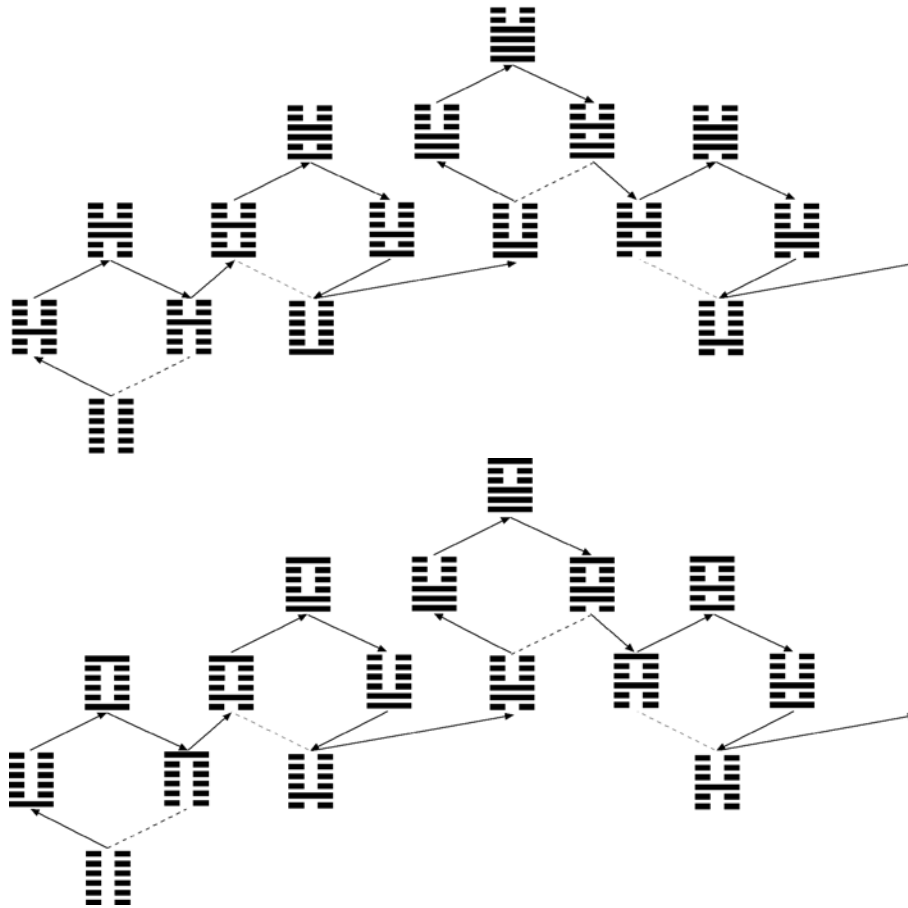


Figure 4: Two Partial Examples of Planar Unfoldings

We have noted that Receptive is, by definition, the first hexagram in all unfoldings. However, most of the rest of the patterns are quite distinct – notice that the first facet identifies the lines which are in focus in each case, whilst subsequent facets show those changes in developing contexts. In the case of these two unfoldings, Peace appears in the same place in both, but this is not generally true. However, it is the case that Creative always appears in the same place in all unfoldings; in the case of planar unfoldings it is always at the apex of the 11th facet, in the case of cubic unfoldings it is always at the apex of the 5th cube. Thus Receptive and Creative retain their role as the primary axis of energy even when the lattice is unfolded.

Cubic Unfoldings

A cube is a natural way of relating to space and, once its relationship to the lattice of trigrams is understood, it can be used as an effective basis for exploring the structure of the complete hexagram lattice. A cubic unfolding will break the lattice down into eight connected cubes: each individual cube is a three-dimensional structure, and each is connected to the next and previous cubes in the sequence by a collection of higher dimensional links, with the sequence of cubes forming a continuous loop, coiling back to the starting point. Planar unfoldings

appear, wrapped inside the cubic unfolding; pair-wise unfoldings appear wrapped inside the planar unfoldings.

There are 20 possible cubic unfoldings of the whole lattice, corresponding to the 20 hexagrams with three yang lines. Four of these present themselves for immediate consideration, focussing on the four different constituent trigrams that can be identified within a hexagram:

- (i) Considering the bottom trigram: this corresponds to *unf* applied to Peace.
- (ii) Considering the lower nuclear trigram: corresponding to *unf* applied to Duration.
- (iii) Considering the upper nuclear trigram: derived from *unf* applied to Influence.
- (iv) Considering the upper trigram: this corresponds to *unf* applied to Standstill.

As for the planar unfoldings, each of these cubic unfoldings gives rise to a distinct perspective on the relationships between the hexagrams, emphasizing some connections at the expense of others; this is the price of reducing the structures to comprehend them. Note that the cubic unfoldings listed here are the only four cubic unfoldings where the focus of awareness is on a coherent domain (i.e. the lines in focus are adjacent).

Other Unfoldings

We have looked at a number of different unfoldings of the hexagram lattice, all based on the same mathematical definitions of *sub* and *unf*. It is also possible to unfold the lattice into four- and five-dimensional subspaces. However, the utility of such unfoldings is initially less than those shown here. This is because all of the techniques shown above reduce the hexagram to a series of three, or fewer, dimensions, and makes the total structure more comprehensible at the expense of simplification. Higher dimensional unfoldings do not initially provide sufficient simplification to bring the lattice structures within our normal cognitive space. However, once one is familiar with the technique, higher dimensional unfoldings offer a way of extending the visualization. For example, it is possible to construct a four-dimensional unfolding that contains pairs of cubes from a three-dimensional unfolding. Also, five-dimensional unfoldings divide the total lattice into halves, allowing us to focus on all the hexagrams with say, a yin line in the first place and then the complementary set of hexagrams with a yang line in that place.

Comparing Unfoldings

We saw when considering *sub*, that more energy in q gives more complex substructures. Because *unf* is based on *sub*, the same situation arises here: the more energy there is in q , the more complex the substructure components of the unfolding are. Following on from this, the more complex the substructure components in the unfolding are, the fewer of them there will be overall. So, a low energy q gives an unfolding with many, simple substructures, whilst a high energy q gives an unfolding with fewer, more complex substructures.

The following interpretation of this is suggested: if there is no energy in q , that is, if awareness is not focussed on any dimension of change, then one will only see each single situation as it arises, the current moment. In this way, one can engage fully with the situation, but there will be no context to help understanding. The result is that experience unfolds as a linear sequence of situations. As one expands one's awareness to include more dimensions of experience, so the amount of context available for consideration in any given situation increases. Then, experience seems less like a linear sequence of situations and more like a

connected web of situations, unfolding as one progresses. Ultimately, if one were able to maintain simultaneous awareness of all the dimensions of experience, then one would be able to see the complete context of a situation.

What are we to make of this multiplicity of perspectives? Birdwhistell¹³ says that “Shao [Yung’s] presentation is a demonstration that no single perspective can enable one to observe all facets of the universe adequately.” Now we have a mathematical way of representing multiple perspectives directly in the organization of the gua.

Conclusions

It is clear that the Yijing has been understood as being about the entokening of abstractions since early in its history. Successive generations of sages have been reflecting on its nature, and the opening chapters of the *Da Zhuan* speak to this theme directly:¹⁴ Chapter One says “Events are grouped to type. Things are divided into classes.” This is abstraction at work. Further, the symbolic means through which these groupings are described are the gua, and Chapter Two goes on to say “the sages devised the gua so that images could be seen therein.” Wang Bi¹⁵ expands on this in the following way: “since the words are the means to express the images, once one gets the images, he forgets the words, and, since the images are the means to allow us to concentrate on the ideas, once one gets the ideas, he forgets the images.” Thus, we see that the Yijing presents us with a related series of abstractions at different levels, each level serving as the point of entry to the next.

In applying Boolean algebra to the gua, a complex six-dimensional structure is revealed which can be seen as representing the totality of reality. In order to make this structure more comprehensible, ways of unfolding it into sets of simpler structures were explored. These techniques suggest that the act of focusing our awareness on different aspects of the situations we encounter results in different perspectives, different views of the world. If Wang Bi’s analogy is followed through, if the textual components associated with the symbols are descriptions of images, and those images represent ideas, then the structural relationships between the gua should also find some meaning in this context. The role of the Boolean lattice is to define the space in which ideas exist. The structural relationships between the gua then express the relationships between the ideas. These structural relationships create subspaces within the lattice, and are themselves open to a narrative description.

It is important that the new techniques made available through the algebraic approach are in harmony with the cosmological underpinnings of the Yijing. To this end I have explored the philosophical implications of applying Boolean analysis to the gua by comparing some of the formal properties of the resulting structures with ideas from the texts of the *Shou Gua* and *Da Zhuan*. Further, the presence of similar diagrams in collections such as the *Zhouyi Tuishi Dadian* shows that some thought has already moved in this direction.

In conclusion, this paper suggests that the Yijing is constantly developing and expanding. As each new culture encounters and embraces it, it absorbs new aspects whilst retaining its core principles. The current phase, incorporating various formal-structural analyses, arises as a natural step – part of the continuous existence of the book through into the age of computational representations.

Lattice Diagrams

Figure 5: The Complete Hexagram Lattice

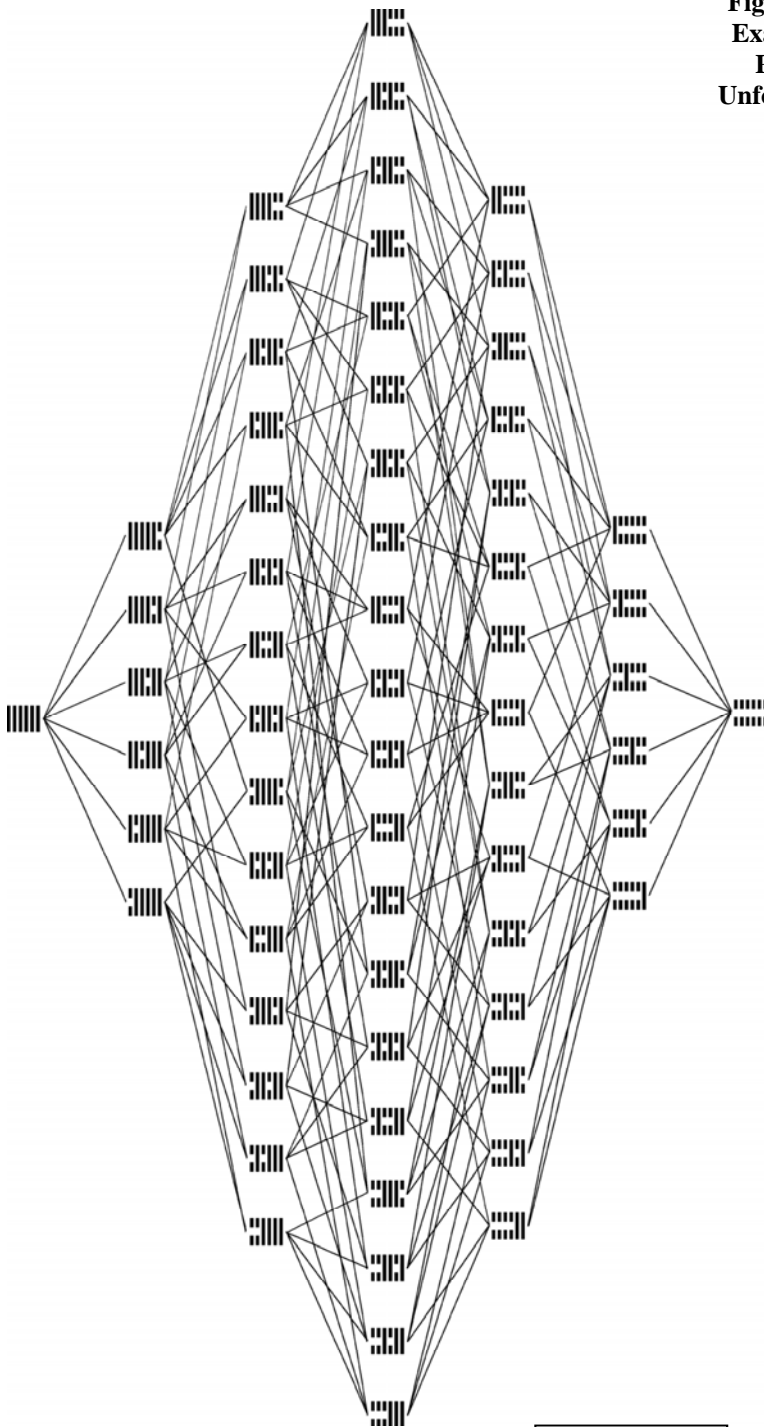
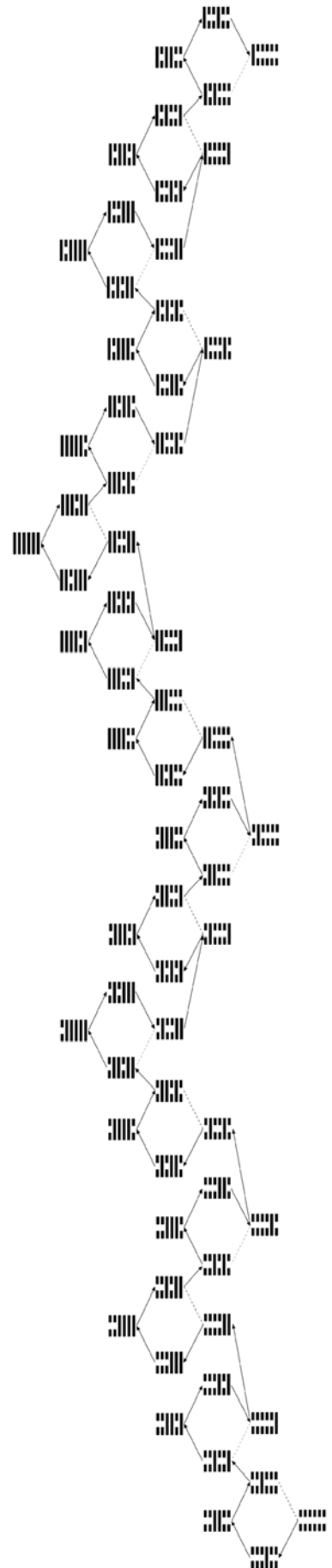


Figure 6:
Example
Planar
Unfolding



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End Notes

¹ Some of the formal background for the work presented here was originally presented in my paper “Boolean Algebra and the Yi Jing” which appeared in *The Oracle: the Journal of Yijing Studies*, Volume 2, Number 7, Summer 1998, pp19–34. This paper is available for download from <http://www.yijing.co.uk>

² See Anne D. Birdwhistell’s book *Transition to Neo-Confucianism: Shao Yung on Knowledge and Symbols of Reality*, published by Stanford University Press, California, 1989.

³ *Zhouyi Tuishi Dadian* “Encyclopaedia of Zhouyi Diagrams” published in two volumes in Beijing in 1994. I am indebted to S. J. Marshall for making scans of some of these diagrams available at <http://www.biroco.com/yijing/scan.htm>.

⁴ Thomas Cleary’s 1989 book *I Ching Mandalas: A Program of Study for the Book of Changes*, published by Shambala Publications Inc. contains a collection of diagrams similar to those in the *Zhouyi Tuishi Dadian*.

⁵ Sung’s work was originally published in 1934 as *Symbols of the Yi King or The Symbols of the Chinese Logic of Changes*. The version that I have seen is from the Paragon Print Reprint Corp. 1969.

⁶ Daniel S. Goldenberg, “The Algebra of the I Ching and its Philosophical Implications”, in *Journal of Chinese Philosophy* 2, 1975 pp149–179. D. Reidel Publishing Company.

⁷ For those readers wishing to pursue the mathematics of lattice work further, a good place to start is *Introduction to Lattices and Order* by B. A. Davey and H. A. Priestly, published by Cambridge University Press, 1987.

⁸ For the sake of familiarity I shall use the English translations of the hexagram names from Richard Wilhelm’s book *I Ching or book of changes*, published by Routledge and Kegan Paul, London, 1983.

⁹ In general, it is possible to represent all of the changes of the Yijing within the different components of the complete lattice. This gives a technique for categorising the Forest of Change, which I plan to explore in a latter paper.

¹⁰ I have developed a range of software tools for exploring and displaying these structures. These can be downloaded from my website at <http://www.yijing.co.uk>

¹¹ The formal definition of sub is: $\text{sub}(p, q) = \{g \in L : p \ \& \ (p \times q) \leq g \leq p \vee (p \times q)\}$

¹² The formal definition of unf is: $\text{unf}(q) = \{\text{sub}(p, q) : p \in \text{sub}(0, \sim q)\}$

¹³ Birdwhistell, *ibid.*, page 96.

¹⁴ Wu Jing-Nuan, *Yi Jing*, published by the Taoist Center, Washington D.C. This quote is from page 263.

¹⁵ Quotes from Wang Bi come from Richard John Lynn’s 1994 translation *The Classic of Changes: A New Translation of the I Ching as Interpreted by Wang Bi*. Published by Columbia University Press, New York.